PROBLEM SOLVING



Mathematics Assessment Project CLASSROOM CHALLENGES A Formative Assessment Lesson

Testing a New Product

Mathematics Assessment Resource Service University of Nottingham & UC Berkeley Beta Version

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Decision Making with Data: Testing a New Product

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to organize, represent and analyze bivariate categorical data in an appropriate way. In particular this unit aims to identify and help students who have difficulty in:

- Choosing a suitable, systematic way to collect and organize the data. This could include constructing tables, two-way tables, or frequency charts.
- Interpreting their own and others' findings clearly and effectively.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics*:

- 1. Make sense of problems and persevere in solving them.
- 3. Construct viable arguments and critique the reasoning of others.
- 7. Model with Mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8-SP: Investigate patterns of association in bivariate data.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding and difficulties. You review their responses and create questions for them to consider when improving their work.
- At the start of the lesson, students reflect on their individual responses and use the questions posed to think of ways to improve their work. They then work collaboratively in small groups to produce, in the form of a poster, a better strategy to the task than they did individually. In the same small groups they implement the strategy.
- In a whole-class discussion students compare and evaluate the different strategies they have used.
- Working in the same small groups, students analyze sample responses to the task.
- In a whole-class discussion, students review the methods they have seen.
- At the end of the lesson or in a follow-up lesson, students reflect individually on their work.

MATERIALS REQUIRED

- Each student will need a copy of the assessment task: *Testing a New Product*, a sheet of paper, and a copy of the *How Did You Work*? review questionnaire.
- Each small group of students will need two large sheets of paper for making a poster, the cut-up sheets *Results of the Survey: Data Sheet 1, 2, 3,* and 4, some felt-tipped pens and copies of the *Sample Responses to Discuss.*
- Provide rulers and graph paper for students who choose to use them.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 100-minute lesson (or two shorter lessons), and 10 minutes in a follow-up lesson (or for homework). Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: Testing a New Product (15 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task: *Testing a New Product*. Introduce them to the context and try to ensure that they all understand the idea of market research when developing a new product.

Imagine that you are part of a research organization and are advising Judi on how to market a new type of deodorant. Her survey is intended to find out how people are influenced by the packaging and fragrance of the deodorant. Read the task carefully and then answer

Show the students the slide P-1, explaining that it shows four further responses. Discuss these briefly and remind the students that Judi has collected forty responses in all. However, also remind them that their task today is to plan how they will organize the data; they are not expected to analyze it at this stage.

the questions.



It is important that, as far as possible, students are allowed to answer the questions without assistance. Some students may find it difficult to get started: be aware that if you offer help too quickly, students will merely do what you say and will not think for themselves. If, after several minutes, students are still struggling, try to help them understand what is required.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, reassure them that they will have time to revisit and revise their solutions later.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. We suggest that you do not score students' work. Research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students at the start of the next lesson.

Common issues	Suggested questions and prompts
Student questions are inappropriate For example: The student writes questions that cannot be answered using the given data. OR: The student writes questions such as 'Which product do people prefer?' or 'Which fragrance do people prefer?'.	What data is being collected?What is the purpose of the survey?How do your questions help Judi?
Student description of how the data could be represented lacks detail or is incoherent For example, the suggested representation does not separate males and females.	 You need to provide more details about how you plan to represent the data. Support your description with an example. Write your description as if you are instructing someone to carry out the task.
Student has not understood the purpose of the survey For example: The student focuses on the most popular product rather than what influences people when purchasing a deodorant.	 Does your description of how to represent the data fulfill the purpose of the survey? Why do you think Judi did not ask people the two questions: Which name do you prefer? Which fragrance do you prefer? What decisions have you made when representing the data? Justify these decisions.
Student representation does not include all the data	• Explain why your representation does not include all the data.
Student representation is such that the data is 'under-summarized' For example: The student creates a table of all the results for the four options but has not shown how they will make any decisions from this. (For example, using mean ratings).	 How will Judi be able to make a decision about what people prefer using your representation? How can you further summarize the data? How can you represent the pictures as number ratings? How can you use these ratings to summarize the data for each option?
Student does not explain how the representation will be helpfulFor example: The student does not explain why they have combined male and female data.Or: The student does not explain why they have represented the data as charts.	What decisions did you make when representing the data? Justify these decisions.How will your representation help Judi?

SUGGESTED LESSON OUTLINE

Because the description of Judi's survey is brief and the amount of data collected is limited, the results may be ambiguous. This ambiguity is discussed towards the end of the lesson.

Review individual solutions to the problem (10 minutes)

Return the assessment task, along with a sheet of paper, to the students

If you did not add questions to individual pieces of work, write your list of questions on the board. Students are to select questions appropriate to their own work, and spend a few minutes thinking about them.

Begin the lesson by briefly reintroducing the problem.

Recall the task: Testing a New Product. What was it about? Today you are going to work together to try to improve your initial responses to this task. I have looked at your work and I have some questions I would like you to think about. On your own, carefully read through the questions I have written. I would like you to use the questions to help you to think about ways of improving your own work. Make a note of anything you think will help to improve your work.

Collaborative small-group work (25 minutes)

Organize the class into groups of two or three students.

Give each group a large sheet of paper and some felt-tipped pens.

Would two or three people like to share the questions they wrote down? What could you find out from the data that would be useful to Judi? [Write two or three responses to this on the board.]

In your groups you need to write down the questions you will plan to answer.

Deciding on a Strategy

Invite students to plan what they are going to do:

In a short while I am going to give you all the data from Judi's survey. Before I do this, I want you to plan carefully what you will do with it! I want you to share your ideas and plan a joint method for organizing and analyzing the data.

Slide P-2 of the projector resource, Planning a Joint Method, summarizes the activity:

Planning a Joint Method

- 1. Take turns to explain your ideas for a method.
- 2. Listen carefully to each other and ask questions if you don't understand or agree.
- 3. When everyone in the group has explained their ideas, try to agree on a joint method that is better than all of them.
- 4. Make sure that everyone in your group can explain your chosen method.
- 5. Write a brief outline of your planned method on your large sheet of paper.

Implementing the Strategy

Give each group another large sheet of paper and the cut-up cards, *Results of the Survey: Data Sheet 1, 2, 3,* and *4*.

Turn over your large sheet of paper and on the other side try to use your method to analyze the data.

State on your poster how your choice of method will help Judi.

You need to make a clear recommendation to Judi about which fragrance and which packaging she should use.

While students work in small groups, you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches to the task

Listen and watch students carefully. Note different approaches to the task. In particular, note any common difficulties. Are the students able to consider the task from Judi's viewpoint? How do the students coordinate the three variables: fragrance, packaging and gender? How do they organize the data; do they use tally charts or two-way tables? Do the students use all the data? Are students aware of decisions they are making? Do students check their totals? What math do students use on the data? You can then use this information to focus a whole-class discussion towards the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that encourage students to clarify their own thinking. In particular focus on the strategies rather than the solution. Encourage students to justify their statements.

What is your method? What do you need to do next? What decisions have you made? Why have you made these decisions? Will it be clear to Judi whether people are influenced more by the packaging or by the fragrance? How can you check your work?

You may want to use the questions and prompts in the *Common issues* table to support your questioning. If the whole class is struggling on the same issue, you could write one or two relevant questions on the board or hold a brief whole-class discussion.

Sharing different approaches (10 minutes)

Hold a whole-class discussion on the strategies used to produce a group solution. Ask two or three groups of students with contrasting approaches to present their posters and describe the approach used, rather than the results obtained. Encourage the rest of the class to ask questions or explain a strategy in their own words. As part of their presentation, ask students:

What approach did you use? How was this strategy different to your individual approach? How did looking at other strategies in your group influence your thinking?

Did anyone use a similar/different strategy to this one?

This will help students with the next activity where they will be critiquing two different approaches to the task.

If you plan to take two lessons for the task, then end the first lesson here. If time allows, choose some groups to share their strategies. Choose groups who have adopted different strategies.

Collaborative analysis of *Sample Responses to Discuss* (15 minutes)

Distribute copies of the Sample Responses to Discuss to each group of students.

This task gives students an opportunity to consider different ways of approaching the task, without providing a complete solution strategy.

First, allow students a little time to familiarize themselves with one of the sample responses:

Each person should choose a different sample response to look at.

Take a moment individually to read through the sample student response and think about what the student is doing.

All the students have counted the data correctly, so you don't need to check this.

There are some questions for you to answer as you look at the work. Write down the answers on the sheet. You may want to add notes to the work to make it easier to follow.

If you don't understand everything, don't worry, because you will have an opportunity to discuss it together afterwards.

When students have had time to read the sample responses individually, invite them to discuss together:

In your groups you are now going to share your thoughts.

Slide P-3 of the projector resource, *Sample Responses to Discuss*, describes how students should work together.



During the small group work, support the students as in the first collaborative activity. Also, check to see which of the methods students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the collaborative group work.

What decisions has the student made? What do you think of these decisions?

Penny collates the data into a tally table. She represents the pictures by numbers and uses them to calculate a total rating for each deodorant. Penny has combined male and female data. She should explain why she has done this.

I will organize the cardo into a table. Each picture rating will have a score 0, 1, 2, 3 or 4.									
	Ø	B				TOTAL			
Bouquet A	ш	11111	++++ +++	**** ***	++++	3×0+7×1+13×2+12×3 +5×4			
			111	111	1	mt = 8 9			
Bouquet B	III	++++-111	HH+ 11	****	HII- II II	3×0+8×1+7×2+13×3			
				111		49×4= 97			
HunkerA		++++	HIF HII	++++- ++++	1111 -101 1				
			l III I	1111 .					
Hunker B	١	1117	++++ 111	++++ ++++	411F WFF				
				++++	<u>u </u>				

Does it make sense to use a rating of zero?

Do the calculations not take into account the people who thought the deodorant was terrible? [Having a rating of zero is fine, but Penny should justify this decision]

Penny could put the totals into a two-way table.

Students may conclude that people prefer:

- fragrance B to A
- Hunter packaging to Bouquet packaging.

This may be useful to Judi, but does not answer her key concern, which is how people are influenced by fragrance and packaging.

She could also conclude, from her analysis, that people are slightly more influenced by fragrance than packaging. This conclusion is fragile as there are many other factors that may influence people.

A disadvantage of Penny's method is that she has ignored gender, so she cannot draw any conclusions about whether there is a difference between male and female responses. Completed two-way table:

	Α	В
Bouquet	89	97
Hunter	107	113

Harry has coded each picture. He has taken gender into account. His analysis (so far) suggests that Bouquet A is more popular with females than males.

Harry's method is inefficient. He could have organized the data into a tally chart for each code.

It will be difficult for Judi to make a decision when comparing four charts.

On the other hand, his method will allow Judi to make comparisons across all variables.





Whole-class discussion: Comparing different approaches (20 minutes)

Hold a whole-class discussion to consider the different approaches used in the sample work. Focus the discussion on parts of the task students found difficult. Ask the students to compare the different solution methods. First project Slide P-4, Penny's work.

What has Penny done?

What do like about her approach? How would her analysis be useful to Judi? What are the disadvantages of her approach?

Now repeat with Harry's work, using Slide P-5 and a similar set of questions.

Students may spontaneously begin to compare the two approaches. If not, explain that you want them to compare the two approaches. Use Slide P-6 and the questions below to focus the class discussion.

Which approach would be more useful to Judi? Why? Which approach did you find most difficult to understand? Why?

To support the discussion, you may want to use Slide P-6 of the projector resource.

Collaborative review of the survey (10 minutes)

During this activity students have the opportunity to critique and improve Judi's survey.

In your groups spend some time critiquing Judi's survey and thinking of ways to improve it. What is unclear about Judi's survey? Has Judi made any assumptions?

Judi's survey is limited. She has not collected enough data to make meaningful conclusions and the description of the survey lacks clarity. For example, it is not clear how she selected the 40 participants. She does not explain to participants whether they should think about a purchase for themselves, their partner, son, daughter or next-door neighbor? Judi assumes only the packaging and the fragrance need to be considered. However, there are many factors that influence a customer's choice of deodorant, such as price. Another weakness of the survey could be that customers very rarely smell a deodorant in a shop. This may imply that Judi is concerned about returning customers (who have had chance to smell the deodorant) rather than new customers, but this is not clear. Judi may also be interested in what it is about the packaging that influences people; the color, the wording, shape etc.

Whole-class discussion: Reviewing the survey (10 minutes)

Hold a brief whole-class review of Judi's survey.

What assumptions has Judi made? What questions would you like to ask Judi? How does this affect conclusions made from the results of the survey?

Follow-up lesson (or possible homework): Individual reflection (10 minutes)

Give out the sheet *How Did You Work?* and ask students to complete this questionnaire. The questionnaire should help students review their progress.

Think carefully about your work this lesson and the different methods you have seen and used. Spend a few minutes on your own answering the questions.

If you have time, ask your students to read through their original solutions again and, using what they have learned, have another go at completing the task. In this case, give each student a blank copy of the assessment: *Testing a New Product*.

SOLUTIONS

Assessment task: Testing a New Product

1. Students need to create questions that will be useful to Judi. Suitable ones are:

- What packaging do males / females / people prefer?
- What fragrance do males / females / people prefer?
- Are males / females / people more influenced by fragrance or by packaging?

2. This question is about planning and student responses will depend on their answers to the first question. A good response would include an explanation of how they will collate the data, how they will analyze it, and how they will present their findings so that Judi can make sense of their work.

The main challenge to students is deciding on how to organize and represent data that has three variables; gender, fragrance and packaging. There are many ways students could approach this task. Below are some possible examples.

Some students may decide to distinguish between male and female choices, while others may combine the figures. What is important is that they are aware of their decisions and justify them.

One approach is to use numerical ratings and means (or totals), as Penny did in the same work. For example: 0 = strong dislike; 1 = dislike, 2 = neutral, 3 = like, 4 = like a lot.

Classwork task: Analyzing the data, making a recommendation

Note that much of the collaborative group work is about planning. However, students are also asked to analyze the data. Using the numerical rating approach above, the analyzed data would look like this:

Male	0	1	2	3	4	Mean rating
Bouquet A	3	5	6	4	2	37/20
Bouquet B	3	7	5	4	1	33/20
Hunter A		1	6	8	5	57/20
Hunter B	1	2	5	7	5	53/20

Female	0	1	2	3	4	Mean rating
Bouquet A		2	7	8	3	52/20
Bouquet B		1	2	9	8	64/20
Hunter A		4	6	6	4	50/20
Hunter B		2	3	8	7	60/20

Male and female results combined

	0	1	2	3	4	Mean rating
Bouquet A	3	7	13	12	5	89/40
Bouquet B	3	8	7	13	9	97/40
Hunter A	0	5	12	14	5	107/40
Hunter B	1	4	8	15	12	113/40

The means may be more simply displayed using two-way tables:

Males	ales Females Males and fe					d female	8		
	А	В			А	В		А	В
Bouquet	37/20	33/20		Bouquet	52/20	64/20	Bouquet	89/40	97/40
Hunter	57/20	53/20		Hunter	50/20	60/20	Hunter	107/40	113/40

The tables suggest that when purchasing a deodorant, males are influenced more by the packaging than the fragrance whereas females are influenced more by the fragrance. When the data for males and females are combined the results are less clear.

The pictorial ratings could also be represented by different numbers; for example, -2, -1, 0, 1, 2 or 1, 2, 3, 4, 5.

Students may decide to represent the data as a series of charts. These charts may distinguish between male and female, may group the data for each product by its rating or may group the data for each rating by its product. Students may use a description or a number for its rating. Here is one example:



The charts can be used determine whether males and females, when purchasing a deodorant, are influenced more by packaging or by fragrance. The data suggests that males are influenced more by packaging, females by fragrance.

Students are asked to make a clear recommendation to Judi. As the sample sizes are small, this is difficult to do with any confidence. One recommendation might be to concentrate on the female market as their ratings are higher overall, and to go for fragrance B in the Hunter packaging.

The significance or otherwise of these results could of course be tested by more sophisticated methods, but the above level of analysis will be sufficient at this level.

Testing a New Product

Judi is starting a new business making deodorants.

Before producing the deodorants on a large scale, Judi wants to find out how people are influenced by the fragrance and the packaging.

She makes two fragrances, A and B, and two possible ways to package these deodorants, Bouquet and Hunter.

Judi conducts a survey. She asks 40 people to comment on the packaging and sample a small amount of deodorant from four different containers:



The people are not told that 'Bouquet A' and 'Hunter A' both contain exactly the same fragrance, A, or that 'Bouquet B' and Hunter B' contain the same fragrance, B.

Each person is asked to fill in a card, checking a box to show how he or she feels about each of the four deodorants. For example, the person who filled in the card shown here is male, hates 'Bouquet A', thinks 'Bouquet B' is wonderful, thinks 'Hunter A' is average and thinks 'Hunter B' is quite nice.

Sex M			
L.			
Bouquet A	\checkmark		т.
Bouquet B			\checkmark
Hunter A		\checkmark	
Hunter B			

Judi wants you to analyze all the data.

Remember that Judi wants to find out how people are influenced by the fragrance and the packaging. Before you do this, plan your method.

1. What questions may be answered by examining and reviewing data collected on the survey cards?

2. Describe clearly how you will organize, display and analyze the 40 responses in order to answer your questions. It may help if you make up a few results using the blank data sheets provided.



Blank Data Sheets



Hunter B

Hunter B













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Bouquet A				
Bouquet B		~		
Hunter A			/	
				/























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Bouquet A	-		\checkmark
Bouquet B		V	
Hunter A			V
Hunter B		1	



















Sample Student Work: Penny

I will organize the cards into a table. Each picture rating will have a score 0, 1, 2, 3 or 4.							
	\bigcirc		2		$\left \begin{array}{c} \bigcirc \\ 4 \end{array} \right $	TOTAL	
Bouquet A	111	++++ 1	++++ ++++ 	++++- +++ 	1 -11+++-	$3 \times 0 + 7 \times 1 + 13 \times 2 + 12 \times 3 - 5 \times 4$ M = 89	
Bouquet B	111	++++-111	#### 11	++++-++++ 	++++- () () (3×0+8×1+7×2+12×3 +9×4=97	
HunterA		++++-	+++- ++++	++++- ++++	1111- (() (
Hunler B	١	1117	1 ++++ «1	++++ ++++ 	1111 H11		

Complete the rows at the bottom of Penny's work.

What conclusions could Penny draw based on her analysis? Explain your answer.

Are there any disadvantages to Penny's method? What improvements would you suggest?

Sample Student Work: Harry



Harry plans to graph the data for Bouquet B, Hunter A and Hunter B in the same way.

Complete the last column of Harry's graph.

What conclusions could Harry draw based on his analysis? Explain your answer.

Are there any disadvantages to Harry's method? What improvements would you suggest?

How Did You Work?

Tick the boxes, circle an option and complete the sentences that apply to your work.

1.	Our solution is similar to one of the sample responses	OR	Our solution is different from both the sample responses						
	Our solution is similar to (add name of the student)		Our solution is different from all of the sample responses						
	I prefer our solution / the student's solution (<i>circle</i>)		because						
	This is because								
2.	Now that you have seen Penny's and Harry's work, what would y	vou do i	if you started the task again?						
2.	2. What advice would you give a student new to this task to help them with difficulties?								
4.	How could Judi's survey be improved?								

Student Materials

Examples of Responses









Planning a Joint Method

- 1. Take turns to explain your ideas for a method.
- 2. Listen carefully to each other and ask questions if you don't understand or agree.
- 3. When everyone in the group has explained their ideas, try to agree on a joint method that is better than all of them.
- 4. Make sure that everyone in your group can explain your chosen method.
- 5. Write a brief outline of your planned method on your large sheet of paper.

Sample Responses to Discuss

- 1. Take turns to explain to the rest of the group what the student in your sample response has done and what your answers are to the questions on the sheet.
- 2. Listen carefully to the explanations. Ask questions if you don't understand.
- 3. When everyone is satisfied with the explanations, you may want to change some of your own ideas!

Sample Student Work: Penny

I will o Each p	nganize	e the a	ards in XII hau	to at re a s	able. core 0	9, 1, 2, 3 or 4.
	0		2			TOTAL
Bouquet A	111	++++ 11	\ 	+ ++++ ++++ 1	-++++	$3 \times 0 + 7 \times 1 + 13 \times 2 + 12 \times 3 + 5 \times 4$ M = 89
Bouquet B	111	++++- 1	HH+ 11	++++-++++ 	++++- (1 / (3×0+8×1+7×2+13×3 +9×4= 97
Hunter A		\\ }	 - 	++++- ++++ ((\{}+ (()	
Hunter B	١	1117	1111 111.	//// //// ////	111 H11	

Sample Student Work: Harry



Compare Strategies

I will organize the cardo into a table. Each picture rating will have a score 0, 1, 2, 3 or 4.												
	$\overline{0}$		2	(: 3		TOTAL						
Bouquet A	111	1111	\ 	++++ ++++	-++++	$3 \times 0 + 7 \times 1 + 13 \times 2 + 12 \times 3 + 5 \times 4$ M = 89						
Bouquet B	111	++++-111	HH+ 11	\\\+ \\\	+111- 11 1 (3×0+8×1+7×2+13×3 +9×4=97						
HunterA		++++	++++- \+ +++ 11	++++- ++++ ((1111 (() (
Hunler B	١	1111	++++	++++ ++++	444 444							

Harry

I will code each tick
$$H = Hornble$$
, $N = Naoty$, $O = Ok$,
 $L = Hornble$, $W = Wonderful$
Bouquet A
Makes
 $H, N, O, L, H, H, Q O, O, L, W, N, O, W$
 $W, N, L, W, N, N, O, H, L, N, L, L, O, O, L, W, O$
 L, N, O, O, L, O, L, L



Penny

Mathematics Assessment Project CLASSROOM CHALLENGES

This lesson was designed and developed by the Shell Center Team at the University of Nottingham Malcolm Swan, Clare Dawson, Sheila Evans, Marie Joubert and Colin Foster with Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by David Foster, Mary Bouck, and Diane Schaefer

based on their observation of trials in US classrooms along with comments from teachers and other users.

This project was conceived and directed for MARS: Mathematics Assessment Resource Service

by

Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan

and based at the University of California, Berkeley

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