Assessment Task Exemplars

Here we set out to show something of the range of tasks designed and developed by the Shell Centre for Mathematical Education for various projects in the US and the UK over the last 25 years.

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These tasks were chosen to illustrate a diversity of styles, methodology and topics, including:

- Computer-delivered and traditional tasks
- Medium length (10 minutes) to extended (several lessons)
- Group work or individual work
- A variety of contexts, including pure mathematics, science, design, planning and finance
- Tasks designed to assess the regular mathematics curriculum, others with a focus on cross-curricular problem solving and scientific reasoning
**Animals**

This problem gives you the chance to:
- find the median, mode and range of a set of tabulated data
- interpret graphs

Hugh works in a nature reserve. He has caught a number of animals of one species in an area of woodland. He records their weights in grams, then he puts their weights in grouped frequency tables.

<table>
<thead>
<tr>
<th>Weight in grams</th>
<th>Number of animals</th>
<th>Weight in grams</th>
<th>Number of animals</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 29</td>
<td>1</td>
<td>20 - 29</td>
<td>3</td>
</tr>
<tr>
<td>30 - 39</td>
<td>4</td>
<td>30 - 39</td>
<td>8</td>
</tr>
<tr>
<td>40 - 49</td>
<td>8</td>
<td>40 - 49</td>
<td>5</td>
</tr>
<tr>
<td>50 - 59</td>
<td>10</td>
<td>50 - 59</td>
<td>2</td>
</tr>
<tr>
<td>60 - 69</td>
<td>7</td>
<td>60 - 69</td>
<td>4</td>
</tr>
<tr>
<td>70 - 79</td>
<td>5</td>
<td>70 - 79</td>
<td>7</td>
</tr>
<tr>
<td>80 - 89</td>
<td>2</td>
<td>80 - 89</td>
<td>5</td>
</tr>
</tbody>
</table>

Hugh uses these tables to plot the two graphs shown below.

1. Find the median weights of the male and female animals. In which groups do they lie?

2. Using the graphs and tables for evidence, write three different statements comparing the similarities and differences between the weights of the male and female animals.
1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you work out your answer.
3. How would you calculate the number of cubes needed for a tower n cubes high?
1. Describe how the width, the span and the thickness of the plank bridge affect the maximum weight it will support. Use words, tables, graphs and formulae.

2. Use your results to predict how much weight the following bridges will support:
   a) thickness = 4 cm  width = 50 cm  span = 2 m
   b) thickness = 12 cm  width = 20 cm  span = 1 m
   c) thickness = 2 cm  width = 30 cm  span = 6 m
   d) thickness = 12 cm  width = 20 cm  span = 10 m

The bridge strength calculator won’t tell you!
Sample student report

- First, I discovered that every centimetre in width (if I kept span and thickness the same) the maximum weight went up 5 N each time. That meant that the weight must be multiplied by 5.

Secondly, I discovered that if I kept width and span the same, and changed thickness, there was no particular pattern. So I used my formulae so far to discover what I needed to do to get the correct answer. Then I discovered, if thickness was squared, and timed by Sw, the maximum weight matched. The formulae was now: \(5W \times T^2\)

Thirdly, I kept width and thickness the same and change span. I discovered that the numbers got smaller. Then I found out you had to divide span by the existing formula. The final formula is: \(\frac{5W \times T^2}{S}\)
You have been asked to design a sports bag.

- The length of the bag will be 60 cm.
- The bag will have circular ends of diameter 25 cm.
- The main body of the bag will be made from 3 pieces of material; a piece for the curved body, and the two circular end pieces.
- Each piece will need to have an extra 2 cm all around it for a seam, so that the pieces may be stitched together.

1. Make a sketch of the pieces you will need to cut out for the body of the bag. Your sketch does not have to be to scale. On your sketch, show all the measurements you will need.

2. You are going to make one of these bags from a roll of cloth 1 metre wide. What is the shortest length that you need to cut from the roll for the bag? Describe, using words and sketches, how you arrive at your answer.
CONSECUTIVE SUMS

The number 15 can be written as the sum of consecutive whole numbers in exactly three different ways:

\[ 15 = 7 + 8 \]
\[ 15 = 1 + 2 + 3 + 4 + 5 \]
\[ 15 = 4 + 5 + 6 \]

The number 9 can be written as the sum of consecutive whole numbers in two ways:

\[ 9 = 2 + 3 + 4 \]
\[ 9 = 4 + 5 \]

The number 16 cannot be written as a consecutive sum.

Now look at other numbers and find out all you can about writing them as sums of consecutive whole numbers.

Write an account of your investigation. If you find any patterns in your results, then try to explain them fully.

Which numbers can not be written as the sum of consecutive whole numbers?
1. Last Sunday an accident caused a traffic jam 12 miles long on a two lane motorway. How many cars do you think were in the traffic jam? Explain your thinking and show all your calculations. Write down any assumptions you make.

2. When the accident was cleared, the cars drove away from the front, one car every two seconds. Estimate how long it took before the last car moved.
Max has received this email. It describes a scheme for making money.

From: A Crook
Date: Thursday 15th January 2009
To: B Careful
Subject: Get rich quick!

Dear friend,

Do you want to get rich quick? Just follow the instructions carefully below and you may never need to work again:

1. At the bottom of this email there are 8 names and addresses. Send $5 to the name at the top of this list.
2. Delete that name and add your own name and address at the bottom of the list.
3. Send this email to 5 new friends.

1. If that process goes as planned, how much money would be sent to Max?
2. What could possibly go wrong? Explain your answer clearly.
3. Why do they make Ponzi schemes like this illegal?
PERSPECTIVES

Simon is standing on the deck of a ship. To the south he sees four mountain peaks; Mounts Axe, Bob, Col and Don. The diagrams below show what Simon sees.

A guidebook lists the heights of these Mountains as:

- Mount Don: 3,000m
- Mount Bob: 2,000m
- Mount Axe: 2,000m
- Mount Col: 1,000m

1. Which mountain is furthest away from Simon? Explain how you know.

2. Which mountain is nearest to Simon? Explain how you know.

To the north Simon can see two mountains, Mount Bart and Mount Homer. Mount Bart is 20 km away and 1,200m high, Mount Homer is 30 km away and 1,500m high.

3. Which mountain looks taller to Simon? Explain how you know.
POLLUTED RIVER

A factory is situated on the bank of a river. It pours toxic waste into the river. The graphs below show how the waste affects plant and animal life in the river. The animals are tiny creatures which live by eating the plants.

![Graphs showing pollution, animals, and plants downstream from the factory.]

Explain fully what is happening to the numbers of animals and plants in the river as you go downstream. Use the graphs to suggest reasons for these changes.
Imagine you live in this small town. In this town, the number of people injured or killed on the roads has increased steadily over the past four years. Each accident location is shown by a red dot on the map.

Your job is to investigate what could be done about this worrying situation. You have a budget of £100,000 to spend on reducing road deaths and injuries. Your job is to plan the most effective way of spending the money.

Some possible ways of spending the money are shown on the next two pages.

Use the computer software to explore the data and discover possible causes of the accidents. Write and present a convincing group report, showing graphs and tables to support your argument.
### Possible measures for improving road safety

<table>
<thead>
<tr>
<th>Measure</th>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Road Safety campaign</strong></td>
<td>A poster and leaflet campaign can be effective when it targets a particular cause of accidents. You will need to describe • the focus of the campaign; • the time of year it will appear; • the type of person it will target. You need to renew the campaign each year for it to continue having an effect.</td>
<td>£20,000 per year</td>
</tr>
<tr>
<td><strong>Traffic lights</strong></td>
<td>Traffic lights can control the flow of traffic at junctions or other hazards, stopping some traffic while other traffic is allowed to go.</td>
<td>£30,000 per junction</td>
</tr>
<tr>
<td><strong>Mini roundabout</strong></td>
<td>Mini-roundabouts are often only marked out with white paint. They are used on roads that have an average speed of 30mph or less. They are often used to reduce speed before a series of road humps.</td>
<td>£10,000</td>
</tr>
<tr>
<td><strong>Large roundabout</strong></td>
<td>Large roundabouts allow one stream to give way to the other. When they are they are called chicanes.</td>
<td></td>
</tr>
<tr>
<td><strong>Pelican crossing</strong></td>
<td>Pelican crossings control vehicle and pedestrian movements with traffic lights. Pedestrians must wait for the “green man” before crossing the road.</td>
<td>£18,000</td>
</tr>
<tr>
<td><strong>Cycle lane</strong></td>
<td>Cycle lanes help keep bikes separate from other road users. They can be either on the side of the road or off-road.</td>
<td>£60 per metre</td>
</tr>
<tr>
<td><strong>Traffic island and pedestrian refuge</strong></td>
<td>Traffic islands in the centre of a road to help reduce vehicle speeds and stop overtaking. If it includes a gap in the middle of the island it is called a refuge; it allows pedestrians to cross half the road at a time.</td>
<td>£3,000</td>
</tr>
<tr>
<td><strong>Speed camera</strong></td>
<td>Speed cameras automatically photograph the number plates of drivers exceeding the speed limit. Many speeding drivers have been convicted by the photographic evidence.</td>
<td>£25,000</td>
</tr>
<tr>
<td><strong>Speed humps</strong></td>
<td>Road humps can only be put on roads with speed limits of 30 mph or less. A series of humps should be about 50 metres apart and have a speed reducing feature at both ends, such as a road narrowing or mini roundabout.</td>
<td>£1,000 per hump</td>
</tr>
<tr>
<td><strong>School crossing patrol</strong></td>
<td>A lollipop lady can help to ensure the safety of younger children. It is helpful if approaching traffic is slowed down by other measures.</td>
<td>£5,000 per year</td>
</tr>
</tbody>
</table>
SMOKING

The chart below resulted from a study of the smoking habits of men. It shows data for about 1,000 men in each of four categories: non-smokers, and those who smoke 1 to 9, 10 to 39, or more than 40 cigarettes a day. It shows how many men would be expected to survive to each age. For example, of 1,000 men aged 25 who do smoke more than 40 cigarettes per day, about 856 will survive to the age of 50.

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of survivors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zero</td>
</tr>
<tr>
<td>25</td>
<td>1,000</td>
</tr>
<tr>
<td>30</td>
<td>994</td>
</tr>
<tr>
<td>35</td>
<td>987</td>
</tr>
<tr>
<td>40</td>
<td>978</td>
</tr>
<tr>
<td>45</td>
<td>964</td>
</tr>
<tr>
<td>50</td>
<td>944</td>
</tr>
<tr>
<td>55</td>
<td>909</td>
</tr>
<tr>
<td>60</td>
<td>855</td>
</tr>
<tr>
<td>65</td>
<td>777</td>
</tr>
<tr>
<td>70</td>
<td>667</td>
</tr>
</tbody>
</table>


Use the data in the table to write comments on the following four opinions. You should try to reply to each statement as fully and informatively as possible.

a) I am 25 years old. I only smoke 5 cigarettes per day. Smoking isn't going to affect me much at all.

b) I am also 25. I am a heavy smoker (about 50 per day). I reckon that I might reduce my lifespan by two or three years, but it's not that much really.

c) I am 45 and smoke about 20 per day. I guess I stand about a 70% chance of reaching the age of 70. That is little different to a non-smoker.

d) This table alone proves that smoking is a cause of early death.
SECURITY CAMERA

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. The camera can turn right round through 360°. The shop owner places the camera at point P, in the corner of the shop.

Plan view of the shop

1. The plan shows ten people are standing in the shop. These are labelled A, B, C, D, E, F, G, H, J, K. Which people cannot be seen by the camera at P?

2. The shopkeeper says that "15% of the shop is hidden from the camera" Show clearly that he is right.

3. (a) Show the best place for the camera, so that it can see as much of the shop as possible.

   (b) Explain how you know that this is the best place for the camera.
**MACHINES**

*(computer-based individual task)*

**Screen 1**

You can use cog wheels to make machines.

Drag some cog wheels onto the bar.

Click on the "Go" button.

The cog wheels will start to turn.

The counters will show how many times each cog wheel has turned.

1. Make a machine with two cog wheels so that when the left cog wheel turns 5 times, the right hand cog wheel turns 15 times.

**Screen 2**

2. Make a machine with three cog wheels so that when the left cog wheel turns 10 times, the right hand cog wheel turns 15 times.

3. Now find a different solution to the problem (still using three cogs wheels).
**Screen 3:**

This time you cannot check your answer by clicking “Go”.  

4. Make a machine with **four** cog wheels so that when the left cog wheel turns 15 times, the right hand cog wheel turns **20** times.

![Image of machine with cogs](image.png)

- 32 teeth
- 8 teeth
- 24 teeth

5. Now find a **different** solution to the problem (still using **four** cog wheels).

In your work book, describe any methods you have used or rules you have found which help you to solve problems like these.

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**Notes:**

On this task, students freely explore the effect of dragging different cogs to make different machines. On the first two screens this is fully interactive. On the third the students have to generalise from their earlier results as this screen prevents them from experimenting. For example, on screen 2 they may obtain:

![Image of machine with cogs](image2.png)

2. Make a machine with **three** cog wheels so that when the left cog wheel turns 10 times, the right hand cog wheel turns 16 times.

- 32 teeth
- 8 teeth
- 24 teeth

3. Now find a **different** solution to the problem (still using **three** cog wheels).
FLY FAST
(computer-based task)

The graphing tool below can be used to show data on different types of bird.

You can compare weights, wing areas and cruising speeds by dragging the labels to the axes.

1. Which two birds may be described as both light and fast? You find the name of a bird by clicking on the point that represents it.

2. Here are some statements about birds:
   * Heavy birds fly more slowly than lighter birds.
   * Birds with greater wing areas fly faster than birds with smaller wing areas.
   * Heavier birds have greater wing areas than lighter birds.
   How valid are these statements? Explain how you know, giving evidence to support each explanation.

3. A scientist wants to find a way of predicting the speed of a bird from its weight and wing area. Which graph is the best for this? Draw a sketch of it.

4. A bird has a weight of 65N and a wing area of 0.60m². What do you think its cruising speed will be in flight?

5. Can you find a rule or formula that will enable you to predict the speed of a bird from its weight and wing area?
Notes:

On this task, students freely explore the effect of dragging labels to the axes. For example, they may obtain:

![Graph 1]

or

![Graph 2]
1. Explore the situation. What conclusions can you draw?

2. Jim and Ann run their own experiments to determine how oxygen production depends on temperature. Ann found little or no effect of temperature on oxygen production. Jim found that a temperature had a large effect. Could they both be right? If not, what mistake might they have made?

3. Jim and Ann discuss how to make the plants produce the most oxygen. Jim suggests that they keep the plants as warm as possible. Ann suggests that they keep the lights on full power. Who has the better idea? What conditions do you think produce the most oxygen?
Notes

On this task, students freely explore the effect of dragging labels to the axes. For example, they may obtain:

Drag one of these labels to the graph axis:

The other variable will appear on the slider below.

Click on the slider and see how the graph changes.

Drag one of these labels to the graph axis:

The other variable will appear on the slider below.

Click on the slider and see how the graph changes.
You have the job of organising a table tennis league.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

Plan how to organise the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.