# Maths of Lotto

[These notes – provided here with permission - are an initial draft being readied for publication within the Maths300 project of Education Services Australia]

Overview | Lesson Plan | Classroom Contributions

## Lesson Plan

| Years: 4-12 | Time: 1-3 lessons | Strands: Data, Chance, Number |
|---|---|---|

### Summary


This is a great fun and productive lesson – easy to get started, based on a real community social issue context, yet it yields many worthwhile avenues of mathematical exploration. It aims to become an anti-gambling lesson. Or at least one designed to show the mathematical realities behind Lotto games. Does the community understand their true chances? If not, then they may be exploitable. It is presented as a whole class game which becomes an

> *"After this lesson one of my students called them 'Looto' games"*

investigation involving probability, combination theory, statistics and working mathematically. A computer simulation of the game allows a problem solving extension, and provides a basis for analysis of the realities of real lotto games.


### Learning outcomes and related concepts

- Probability and expectation
- Combination theory and simple proportion
- Statistics
- Interdisciplinary learning
- Working mathematically and mathematical modelling
- The problem solving strategy of ''*solve a simpler problem*''

### Software contributions to learning outcomes

The computer simulation is designed to support the lesson in five main ways, namely:

- Option 1: a simulation of the basic game - a 'bridge' from concrete to a computer environment
- Option 2: playing many games to see the long-run expected results

- **Option 3: investigating if each pair (or group) has the same chance of being drawn**
- **Option 4: changing the number of marbles in the barrel and the number drawn out**
- **Option 5: a simulation of a real community game**

**The basic game (option 1), is simulated so that students can see the software 'plays the game' just as they did. This builds a trust in the model before it is put into 'high speed'.**

*This option can also function as an **electronic blackboard** to illustrate how the game is played; however I usually prefer to introduce the game physically with blocks.*

**Option 2 plays the basic game any desired number of times so that students initial perceptions can be checked against reality and also checked against the theoretical calculation of 1 in 15.**

**Option 3 keeps track of how many times each pair (or group) is selected to check if they are all equally likely to be drawn**

**Option 4 allows the game to change from 6:2 to 7:3 to 9:2, - any combination up to 9:4**

**Option 5 creates a simulated real context where the real chances of lotto prizes and the flow of money can be explored.**

## Resources required

- **Software is available from the Software Library.**
- **Six numbered blocks (in a box or bag) for the class game**
- **Suitable 'prizes' for the winners – (need about 30)**

## Lesson stages

1. **Discussion of real lotto games and demonstration of the simple 6:2 game**
2. **Estimation of chances in the 6:2 game**
3. **Play 10 games – record results on board**
4. **Analysing game results and checking against perceptions – is there a difference?**
5. **Explaining the real chances – how many pairs (or groups) - are they equally likely?**
6. **Are you superstitious? – the psychology of choices**
7. **Playing and analysing other simple games eg. 7:2, 8:2, 7:3, 5:3, 10:2, ….**
8. **Computer simulation**
9. **Analysing real Lotto games and where the money goes**
10. **Educating the public – do they know their real chances?**

## Lesson notes

### 1. Discussion of real lotto games and demonstration of the simple 6:2 game

[back to lesson stages](#)

*"Lotto Games are in virtually every city in the world – and governments and the organisers collect vast sums of money from them."*

I ask students (briefly )for their existing knowledge of the various Lotto games. I keep this very brief as I don't want to get 'bogged down' in explanations at this stage, but it does often give me insights into their current perceptions.

I was not prying, but one of my students cheerfully volunteered that their family bought multiple tickets a week – obviously in the hope of improving the family finances. I knew this family to be 'struggling' in an economic way.

"Do you think the community understands their true chances of winning a prize? – If not then they may be exploitable, spending more money than is wise!"

*"You have to be in it to win it!"*

but also

*"A fool and their money are soon parted!"*

I'd like to show you the mathematics that lies behind such games – and perhaps you could use this to explain the chances to family and friends."

The real game is quite complex – one version has 45 marbles in the barrel and you select 6 numbers in any game, and if your 6 numbers come out you win 'first prize'.

I'd like to show you the mathematics by playing a much simpler game – called the 6:2 game – there are only 6 marbles in the barrel and you have to choose just two of them to 'win the game'.

***Please draw a playing board – that looks like this and circle the two numbers you choose. For example:***

**Game 1:** 1 2 3 4 5 6

## 2. Estimation of chances in the simpler 6:2 game

back to lesson stages

This is an important section – I very much want to know the range of students initial perceptions – which we will come back to and compare later once we know the real chances

We are about to play this much simpler game – I'd really like to know what you think your chances are – sometimes you will win – sometimes not – suppose we played 100 times – how many times do you think you might win? Please write it down and then come to the board and record your guess – so we can see how close we were after playing this much simpler game.

*I average these – so as a group you are expecting about 15 wins out of every 100 games – that's the same as a winner every 6.6 games.*
*Let's see what happens!*

## 3. Play 10 games – record results on board

back to lesson stages

Here we go – I'd like to play 10 games in total, so we can check the results at the end.

I always get a student to come out – draw the two 'marbles' (one at a time – with a pause between them for 'dramatic effect' - and call them out.

**Pic of student drawing blocks and Stick figure of student head holding the 2 blocks – voice bubble – "It's a 4,…-- and a 6".**

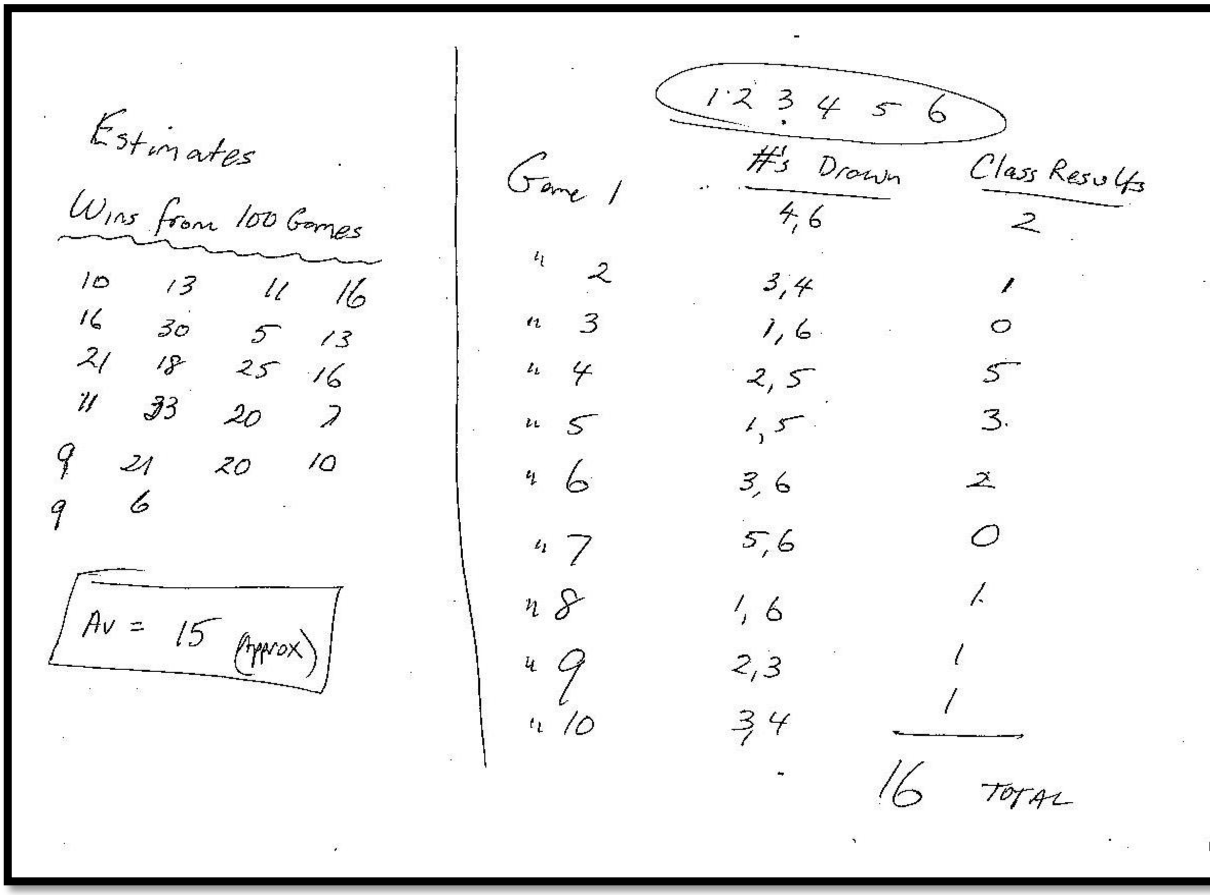
I reward the winners with suitable prizes and record the results on the board:

| | | | | | | | #'s drawn | #class winners |
|---|---|---|---|---|---|---|---|---|
| Game 1: | 1 | 2 | 3 | 4 | 5 | 6 | 4&6 | 2 |

I'd like to play 10 games quickly – so we can get some results to analyse- so set up your game boards.

| | | | | | | | #'s drawn | #class winners |
|---|---|---|---|---|---|---|---|---|
| Game 1: | 1 | 2 | 3 | 4 | 5 | 6 | 4&6 | 2 |
| Game 2: | 1 | 2 | 3 | 4 | 5 | 6 | | |
| Game 3: | 1 | 2 | 3 | 4 | 5 | 6 | | |
| Game 4: | 1 | 2 | 3 | 4 | 5 | 6 | | |
| ………… | | | | | | | | |
| Game 10: | 1 | 2 | 3 | 4 | 5 | 6 | | |

Play the 10 games (as quickly as possible) rewarding the winners and recording the results



Estimates

Wins from 100 Games

10 13 11 16
16 30 5 13
21 18 25 16
11 33 20 7
9 21 20 10
9 6

Av = 15 (Approx)

1 2 3 4 5 6

| Game | #'s Drawn | Class Results |
|---|---|---|
| Game 1 | 4,6 | 2 |
| " 2 | 3,4 | 1 |
| " 3 | 1,6 | 0 |
| " 4 | 2,5 | 5 |
| " 5 | 1,5 | 3 |
| " 6 | 3,6 | 2 |
| " 7 | 5,6 | 0 |
| " 8 | 1,6 | 1 |
| " 9 | 2,3 | 1 |
| " 10 | 3,4 | 1 |
| | | 16 TOTAL |

## 4. Analysing game results and checking against perceptions – is there a difference?

back to lesson stages

**We have played 10 games each – and there are 22 of us, so we have played 220 games as a class.**

**And we have had a total of16 winners.**

**So our chances of a win were 16 out of 220 – which is about 7.3% - i.e we got just over 7 winners for every 100 games.**

**Let's see how that compares to your guesses (expectations).**

**Nearly every time I have tried this the guesses are much more on the optimistic side than reality.**

**"Why do you think your guesses on average are more optimistic than really happens?"**

***I often have several students around the Grade 6 or 7 level writing a guess of 30 or 33 wins out of 100 – they are clearly working on a simple ratio of 2 to 6 – and not the combination theory that really underlies the game – which seems to be one main reason it looks easier than it is – and hence leads people to misjudge their chances and hence are potentially exploitable.***

**Computer support:** I often use the computer at this stage to back up our class results.

I'd like you each to investigate aspects of the game on the computer soon, but at this stage I'd like to demonstrate Options 1 & 2.

Option 1 shows students the computer plays just like we did. Then in Option 2 we can play many games.

Lets' get it to play 220 games- just like we did


We could easily see that the results were similar to our class game (which built trust in the software model) and that from every 100 games we had about 7 winners (much less than the class predictions).

## 5. Explaining the real chances – how many pairs (or groups) - are they equally likely?

back to lesson stages

Here is how a mathematician could explain the chances of a win.

To win the game, your selected pair has to come out of the barrel. So how many pairs are in the barrel?

Students write these down from first principles – some will be systematic, others less so.

**List of all Pairs:**

1,2 1,3 1,4 1,5 1,6

2,3 2,4 2,5 2,6

3,4 3,5 3,6

4,5 4,6

5,6

There are 15 pairs in the barrel – each pair has the same chance as any other – so the chances of a win are '1 out of 15'

1 out of 15 is 6.6% or 6.6 means for every 100 games.

Our results were very close to this!

Option 3 on the computer 'does each pair (or group) have the same chance?' verifies this – it plays many games (try 1,500) and keeps count of how many times each pair is selected

## 6. Are you superstitious? – the psychology of choices

back to lesson stages

A delightful section that made everyone chuckle at how we can be affected in the way we choose our numbers.

All my students readily agreed that each pair has the same chance.

"I wonder if you really believe that? – and we already have the data to prove it!!!!!"

Please look back over the pairs you chose over the 10 games. At any stage did anyone choose the pair 5&6 ? How many times?

I carefully count the number and record it on the board.

Now what about 2 & 5 ?

Here are the results on one Grade 7 class with 22 students in the class. [If 15 pairs (from 220 selections) are distributed 'evenly' then each pair should be selected around 14 to 15 times.

| Pair | Times chosen |
|---|---|
| 5,6 | 4 |
| 2,5 | 23 |

Why are these choices so different? Why have you chosen 2&5 much more often than 5&6? – when you know (or claim to know) them both to have the same chance?

The resultant discussion was fun and fascinating. Students said things like:

"It's because the 2 and 5 are spaced out on the board!"

"It's very unlikely to get two numbers next to each other"

I liked the following explanation- delivered with humour:

"Suppose the 5 is drawn out first (and I model this) – you believe the 6 has 'seen this' and goes and hides in a corner while the '2' jumps up and down saying 'pick me – pick me!' – that's what your guesses suggest

We then considered another couple of pairs. Let's look at 1&2 compared to 2&4. Which do you think will have been selected more often? We checked these with much the same result.

The students could now see clearly what had happened with their guesses and had been influenced by some psychological belief that is illogical.

I then use Option 3 of the software – does every pair have the same chance.

I firstly run 100 games and we see quite a lot of variation – and we looked for such pairs as 2,5 and 5,6. I repeat this several times (by just pressing the space bar for repeat trials.

Now let's try 1,500 trials – since there are 15 pairs and if each pair has the same chance, then each pair should occur about 100 times each.

Draw 2 numbers from 6 numbers

Played 1,500 of 1,500 games

| Numbers | Wins | % Wins |
|---|---|---|
| 1,2 | 121 | 8.1% |
| 1,3 | 88 | 5.9% |
| 1,4 | 112 | 7.5% |
| 1,5 | 97 | 6.5% |
| 1,6 | 101 | 6.7% |
| 2,3 | 83 | 5.5% |
| 2,4 | 91 | 6.1% |
| 2,5 | 103 | 6.9% |
| 2,6 | 78 | 5.2% |
| 3,4 | 89 | 5.9% |
| 3,5 | 103 | 6.9% |
| 3,6 | 93 | 6.2% |
| 4,5 | 126 | 8.4% |
| 4,6 | 109 | 7.3% |
| 5,6 | 106 | 7.1% |

Finished

Translating this to real Lotto – I hold up a typical Lotto card with 45 numbers. Would you ever choose 6 numbers in a row? – of course not they all chorused – that would never happen!!!

However they were all beginning to understand and believe that a group of 6 in a row has exactly the same chance as any other group of 6.

## 7. Playing and analysing other simple games eg. 7:2, 8:2, 7:3, 5:3, 10:2, ….

The following day I asked if they would like to play 'Maths Lotto' again. They all said yes, so I offered two games – the 7:2 game or the 6:3 game.

We'll play whichever one you vote for – so spend a few minutes first working out in which one you have the best chance.

For the 7:2 game, you need to figure out how many pairs are in the barrel. [Answer 21, so the chances are one in 21]

For the 6:3 game you need to figure out how many groups of 3 there are from 6 numbers.[Answer: there are 20, so the 6:3 game is the better (just).

What about  8:2 versus 7:3? Which is the better?

What about 5:3 versus 6:2?

I liked how these challenges encouraged students to work out the chances from first principles.

## 8.. Computer Simulation

**When students have access to computers in small groups, I firstly ask them to revisit Option 1 and 2 to see how the software models the simple game we played.**

**After this, Options 3 and 4 of the software can be an investigative tool to assist the students to investigate and understand any game. i.e change either the number of marbles in the barrel – or the number to be drawn out.**

**Challenges can be such as:**

**1: Explore the games of 6:2, 7:2, 8:2 ... and comment on how changing the number of marbles in the barrel changes the chances.**

**2. Explore the games of 9:2, .9:3, 9:4, 9:5,....9:9 and comment on how changing the number drawn out changes the chances.**

## 9. Analysing real Lotto games and where the money goes.

**After all the earlier explorations and the understandings derived from working out chances from first principles, a common community game such as 45:6 can now be presented.**

**45:6. 45 marbles in the barrel – 6 are drawn out – to win (the top prize) you need to select the correct six.**

**So how many groups of 6 are there in the barrel.**

**If we start to write them down systematically we can get:**

**1 2 3 4 5 6    1 2 3 4 5 7    1 2 3 4 5 8    1 2 3 4 5 9    …….. 1 2 3 4 5 45**

**1 2 3 4 6 7    1 2 3 4 6 8    etc**

**Around about now students realize there will be a ‘LOT’.**

**There are indeed 8,145,060 (which senior students can calculate as 45 C 6.)**

**So the chances, if playing a single game are 1 out of about 8 million.**

**So if you played one game a week you could expect to win - once every 8 million weeks.**

**But a typical ticket allows you to play many games – suppose you play a ticket with 10 games (10 different groups of 6) – you can still expect to win – once every 800,000 weeks – that’s once every 16,000 years!!!**

### Money:

So how much might you win? The game organizers typically take out 40% of all the ticket sales, and distribute the remaining 60% in prizes.

So let's play 10 games a week for 16,000 years.  Assuming a single game costs $1 that is $10 per week

$10 per week for 50 weeks is $500 per year * 16,000 years equals a total outlay of $8 million dollars.

The total prizes you could expect are $4.8 million, (including the first prize), for a nett loss (even though you won!) of $3.2 million dollars.

The above fanciful analysis is based on the fact that 40% of all income is removed from the income, leaving only 60% for distribution of prizes.

## Software Simulation:

Option 5 provides a simulation of a typical lotto game and can provide some realistic data to consider. This allows students to spend any amount per week for any length of time and to simulate what results they could approximately expect.

One major Australian Lotto game has a prize pool of 60% of the income, and of this 60%, 28% goes to the major prize and 72% towards minor prizes.. i.e 16.8% of the total income goes to the major prize and 43.2% of total income goes to minor prizes.

To demonstrate I Click on 'slick 100' and 100 games of 6 are selected at random and placed into the Lotto Games column. That will cost us $60 per week to play these 100 games every week. Or click on slick100 agin to play 200 games per week – or even 1,000 games per week by keep clicking slick 100.

In demo mode the software plays for one week at a time and any prize numbers are highlighted in RED in the games list.

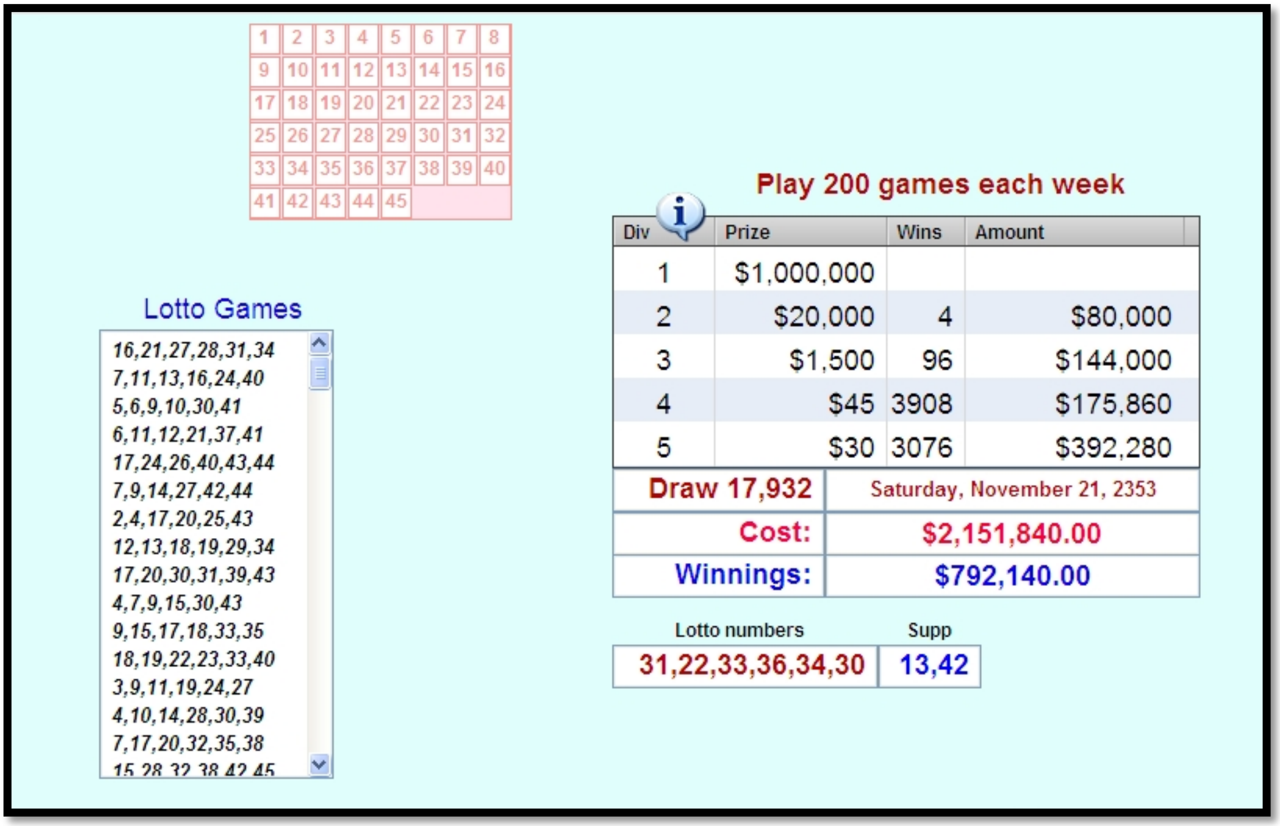
Students can also click on any set of 6 numbers and enter these and then play in Auto mode.

A system entry of choosing 7 numbers will then enter all possible combinations of 6 numbers from these (which will be 7 different groups of 6).

Similarly system 8 will produce 28 different groups of 6 from the 8 numbers.

The software allows up to system 10.

Whichever combination of games is chosen, the software will just keep running, but will STOP if ever a first division prize is won.




The following output shows 200 games per week –and was paused after 17,932 weeks which takes you until the year 2353. At which time over $2 million dollars was invested for a return of nearly $800,000 and a nett loss of approx. $1.3 million

## Student Investigation Challenges:

If students in small groups have access to the software the following are some structured investigations. These are mostly designed to abolish some prevailing ‘myths’ which students (and adult populations) often hold

1. Ask one group to play a single game of 6 numbers in a row – and then another group to choose any 6 numbers they like. Now let the machine run for several thousand games and see if the ‘numbers in a row’ are less likely?
2. Now try system 10 using any 10 numbers in a row (which many students think to be unlikely to win a prize), and another group to choose any 10 numbers sprinkled across the playing board.
3. Choose 6 even numbers versus 3 even and 3 odds
4. Play 1,000 games per week and leave the machine running for ‘many years’.

I found students would creatively set their own investigation agenda. My task as teacher was to encourage them to seriously look at the output and to record what they learned.

In our school we left the machine running overnight – for 'many thousands of years' – and we still had not won the top prize!!!

## 10. Educating the public – do they know their real chances?

This discussion had been going on throughout the investigation – the main message being that the game looks easier to win that it really is – and if people do not understand their real chances they may be misled into spending more than their circumstances warrant.

### One research question:

Students asked family members of friends as a homework exercise: Our research hypothesis was: - "Do people overestimate their chances of winning?'

"In Lotto – someone wins nearly every week – often several winners in the same week. Suppose you were allowed to play 100 games a week – after how many weeks (or years) would you reasonably expect to win the top prize?"

Each student was asked to collect the estimates from at least two citizens. These were compiled on the board and then averaged.

And then compared to the ''real' answer which is:

There are 8,145,060 different groups of 6 numbers. Playing 100 games per week means your chances are 1 out of 81,450. So you can expect to win once every 81,450 weeks – or once every 1,566 years!