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Features in Task Design for Inclusion: An Example of a Mathematical Investigation

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Abstract

Teachers hold beliefs that determine the practices they use to offer accessibility for students; usually taking an approach to start from the easy and known content towards the more difficult and unknown. While this kind of support will enable particular students to engage with these kind of tasks, others might still struggle to start or else find such tasks too easy. In addressing the aforementioned issues, task design for inclusion is discussed within an inquiry-oriented curriculum framework. An inquiry framework posits that irrespective of their academic and learning ability, socioeconomic and cultural backgrounds, all students should have access to and experience a broad mathematics curriculum. Embedded within the principles of representation, action and expression, and engagement of the universal design for learning framework, three task design features are presented. To translate these task design features into practice, an example of a low-floor high-ceiling task is provided. This task, the spiral pattern investigation (see Calleja, 2020), embraces student diversity of ideas and supports an inclusive secondary school mathematics classroom setting. The discussion focuses on how design features can make tasks accessible to students and, hence, how tasks features can allow students to contribute ideas and to engage in higher-order thinking through varied representations of ideas and more complex connections between ideas.

Keywords

Diversity; inclusion; spiral pattern investigation; task design features; UDL principles

Introduction

Inclusive education can be defined as an approach in which all students are respected, can participate actively in learning, and are acknowledged as valuable within the learning community (Moriña, 2017). Prioritising inclusive education implies a commitment, by the learning community, to provide high-quality education that ultimately leads to full participation of all students, irrespective of their diversity (Messiou et al., 2016). Diversity, conceived broadly, comprises different students' capabilities, gender, social and cultural differences. More importantly, in inclusive learning communities, diversity is valued and sought out because it is seen as a benefit rather than a problem to the learning community (Askew, 2015).

While in education there is a greater consensus about students' entitlement to high-quality education and inclusive practices, there still appear to be barriers towards its implementation. Indeed, research suggests that mathematics teachers are still not adequately prepared to foster inclusive classrooms (Bishop, Tan & Barkatsas, 2015; DeSimone & Parmar, 2006). As Askew (2015) argues, within the recurrent traditional view of mathematics teaching and learning, the teaching approach adopted by teachers still seems top-down, transmissive, and intended to direct students to one-size-fits-all outcomes. The teacher, as the knowledgeable person, tends to pass knowledge onto students who act as passive recipients (Gattegno, 1971) because they are perceived to possess limited or no knowledge about the subject matter. This scenario, which is quite widespread (Askew, 2015; Calleja, 2022; Buhagiar & Murphy, 2008), appears to limit teachers' instructional practices and their attempt to include more collaborative and inquiry approaches to the teaching of mathematics (Swan, 2005).

Within outcomes-based teaching scenarios, which see teachers attempting to drive all students towards achieving common learning outcomes, addressing diversity and promoting inclusion appear problematic for teachers (Askew, 2015). This is because, in teaching, the tendency for teachers is to start with the 'known' (e.g., mathematical content) and then gradually move to teach the 'unknown'. Teachers tend to provide students with tasks (such as, problems, questions, and examples) that they perceive as manageable for most students in class. Gradually, teachers provide more challenging tasks based on the belief that learning is a linear and hierarchical process for students. While this approach appears to be inclusive, as teachers choose to offer tasks that students are likely to be able to do, I argue that providing students with achievable or easy tasks is not necessarily inclusive. It is not inclusive because, within any group of students, while some would tend to find a task easy to do, others are still likely to struggle and give up. For the former group, a relatively easy task will exclude them from possibilities to progress and extend their learning. On the other hand, those who struggle may perceive themselves as failures, particularly in view of those students within the same group (or class) who were able to do the same task with little or no effort. This notable difference in students' learning experiences working on what should be an 'achievable' task requires attention from teachers as well as task and curriculum designers.

To address this dilemma, I base my position and argument to design for inclusion by drawing on the work of Roos (2019). She argued that, in education, inclusion is either used as an ideology or as a way of teaching. Moreover, when there is no link to how inclusion as an ideology can be practised in the classroom, translating its meanings and values to the classroom context will be demanding for teachers. In this paper, inclusion is considered in terms of the opportunities that teachers provide so that all students (irrespective of their backgrounds which, amongst others, include gender, ethnicity, culture, and attainment) can participate actively and access the broader mathematics curriculum. The challenging mission for teachers is then to design lessons through which they can facilitate access to mathematics for, and participation by, all students. Additionally, teachers need to have an awareness and ability to act 'just-in-time' so that they can promote, support, and sustain student participation and learning (Calleja, Foster & Hodgen, 2021). This means that, in the mathematics classroom, the quest for inclusion and the implementation of inclusive pedagogies requires teachers to instill an equitable environment where the needs of all students are considered and addressed (Roos, 2019).

Hence, in my attempt to describe and discuss features of inclusion in task design, I seek to illustrate how task designers may embed inclusion as an ideology within task design so that teachers can put into practice inclusive practices centred within an inquiry-oriented mathematics classroom. Within my presentation below of a *spiral pattern* task—intended as an investigation that allows for an open-ended approach to mathematical inquiry (Van Reeuwijk & Wijers, 2004)—I present three task features linked to three key Universal Design for Learning (UDL) principles and illustrate how these can support teachers to promote inclusion in the mathematics classroom.

I start by exploring task features, specifically related to the teaching of mathematical content through problem exploration, and then address teachers' task enactment issues. The argument I make for designing tasks for inclusion is based on the premise that all students should experience a broad view of the mathematics curriculum that includes both the practical and the more abstract content of the subject. The argument builds on the understanding that all students should have the same access to the broader mathematics curriculum in a way that it does not limit their academic potential. Drawing on Askew's (2015) idea of curriculum as inquiry, teaching mathematics would, hence, require teachers to dedicate time for deep learning using tasks that offer opportunities for student discussion, thinking, reasoning, and decision-making.

Designing for Inclusion Through Inquiry

Notwithstanding the efforts that schools make to put their inclusion policies into practice, it is ultimately the teacher who is responsible for designing learning opportunities that are accessible and that cater for the needs of all students (Panizzon, 2015). Accessibility is viewed in the sense that the tasks provided by teachers need to be "accessible to a wide range of students and they extend to high levels" (Boaler, 2016, p. 84) and through which all students can experience success, an achievable mental challenge, and a degree of failure. In the same way, inclusivity is considered at a broader scale and, rather than just dealing with the provision of support, it is conceptualised as welcoming student diversity (Ainscow, 2007) as they engage in inquiry. Hence, tasks that promote inclusivity through inquiry are conceptualised as those that offer both multiple entry points as well as challenges. For challenges to be achievable by all students, task designers need to incorporate skillful thinking and noticing of the support and scaffolding that individual students might need so that they do not give up in frustration while trying to solve the task at hand.

The Context for Designing for Inclusion

Over the past twenty years, in Malta, there has been a restructuring and a reorganisation of the local education system (MFED, 2012; MFED, 2020). For example, the National Curriculum Framework (MFED, 2012) suggested a strategy to improve the quality of education and raise the level of student achievement by moving towards a learning outcomes-based approach (MFED, 2020) intended to provide more flexible learning programmes that support more diverse, integrated, and inclusive learning experiences. These guidelines targeted three key aspects: (1) teaching and learning; (2) assessment; and (3) professional development. While learning programmes encouraged teachers to implement more student-centred teaching using formative assessment practices, professional development was envisioned to provide ongoing opportunities for teachers, rather than the more common sit-and-get sessions provided by outside experts, so that they can work more collaboratively within school learning communities. These changes, involving curricular and policy-oriented reform, were intended to address issues of social justice and equity (Mifsud, 2021). However, according to Borg (2019, 2022), the Maltese education system continues to fail students and particularly those of low socio-economic status who usually end up on the receiving end of the academic underachievement continuum. At secondary level, the practice of ability grouping (also known as setting), the compartmentalisation of topics within the curriculum (presented as a set of isolated, disconnected, fixed, and sequential units of study), transmission and textbook teaching, and the lack of a comprehensive education system potentially has led to social injustices for marginalised students who often lack equal rights and opportunities, differentiated treatment, equal access, and full participation (Borg, 2022). For mathematics, which is one of the core subjects taught in primary and secondary schools (ages 5 to 16 years), this has generally led to transmission teaching (Calleja, 2022; Buhagiar & Murphy, 2008) creating both gender differences and underachievement (Bezzina, 2010).

In Malta, the mathematics curriculum compartmentalises the subject into isolated topics with prescribed content-based syllabi (MFED, 2020) and teachers tend to plan the order of their teaching according to this conceptual layout. In addition, teaching is exam-driven with lessons generally being teacher-dominated and structured into exposition, practice and consolidation phases (Buhagiar & Murphy, 2008). The learning outcomes-based approach (MFED, 2020) has preserved teaching mathematics through transmission with the main resource for teachers being the textbook (Calleja, 2022). While local documents have, to some extent, proposed a more humanised view of school mathematics incorporating a more student-centred and inquiry approach to learning, a more teacher-dominated view of teaching mathematics is still predominantly reinforced through mathematical texts (particularly syllabi, textbooks and examination papers). Such texts present mathematics as definite, often prompting teachers to “seek an authority on the mathematics they present in their classrooms ... reinforced through traditional testing and examinations” (Jaworski, 2010, p. 13). For this reason, discussing design features of the *spiral pattern* task (for example, shifting from having students do what they are told to do by the teacher and moving towards supporting students to take initiative and ask their own questions) is important. It is important as it provides teachers and curriculum designers with a model that can help them conceptualise a more participatory approach in teaching mathematics that helps them to see how they can transform a teacher-dominated approach to teaching to one which is more student-driven. The *spiral pattern* task example is specifically intended to support secondary school teachers in their attempt to move away from being providers of mathematical knowledge towards offering a more inquiry-based, collaborative, participatory, supportive, and inclusive approach to teaching.

The Emergence of Spiral Patterns

In 2007, as part of a master’s degree in mathematics education at the University of Malta, I embarked on an action research project to design a task-based inquiry-oriented secondary school mathematics curriculum based on investigations (Calleja, 2013; Calleja & Buhagiar, 2022). I recall coming across spider webs that could be generated by using patterns to join points on axes (see Appendix for some of the iterations involved in developing the *spiral pattern* task). Initially, I introduced these spirals to stimulate students’ interest in the subject while recognising the beauty of how patterns could create intriguing designs.

For each spiral web, I asked students to pose questions and encouraged them to describe their designs. Students noted that it consisted of lines increasing in length and generating triangles underneath. This spiral pattern offered opportunities for inquiry and exploration. With each question students asked, conjectures were generated. The *spiral pattern* presented here (see also Calleja, 2020) “catches the puzzles and the challenges of mathematical ideas" (Bishop, 1991, p. 115) generated by students while inviting them to 'locate themselves' in the task (Alrø & Skovsmose, 2002). The *spiral pattern* task, thus, is intended to offer the opportunity for all students to use the mathematics they have learned as well as develop new mathematics and mathematical thinking (Van Reeuwijk & Wijers, 2004).

Presenting the Spiral Pattern Task to Students

For this task, which the teacher presents using the task sheet shown in Figure 1, students would need a pencil, ruler and squared (or graph) paper. On a Cartesian grid, using squared (or graph) paper and 1 cm to represent 1 unit, the ***x***-axis and ***y***-axis are drawn from –8 to 8. Next, the set of coordinates (0, 1), (2, 0), (0, –3), (–4, 0) and (0, 5) are plotted, with the sequence of coordinates continued as far as the graph allows. Finally, points are joined by straight lines following the sequence given (see Figure 2).

Figure 1 - Students' task sheet

On your graph or squared paper, use 1 cm to represent 1 unit to draw the ***x***-axis and the ***y***-axis from –8 to 8.

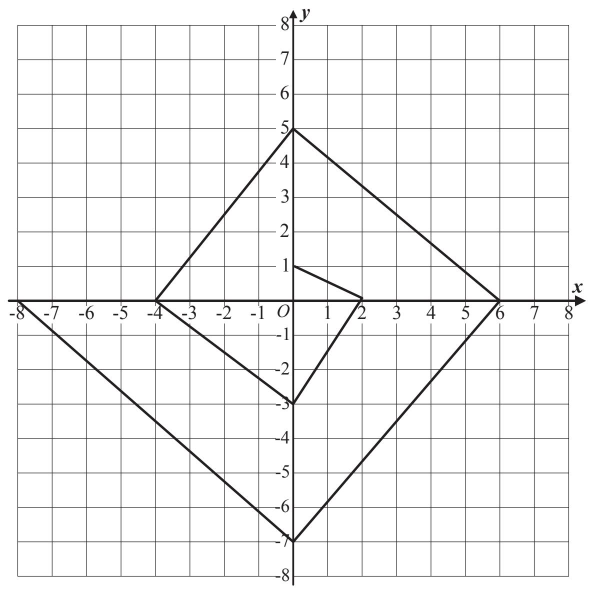
Then plot the points:
(0, 1), (2, 0), (0, –3), (–4, 0) and (0, 5).

Continue this sequence of points as far as you can go.

Using straight lines join the sequence of points that you have.

Look at the emerging spiral design and make statements about what you see, identifying any patterns and links to mathematics you have learned so far.

Figure 2 – The spiral pattern



Once students construct the spiral pattern, the teacher asks them to take notes about what they can see within the resulting pattern. In opening up the task for inquiry, the teacher encourages students to think about anything related to mathematics that they might be able to see or make connections with within the spiral. Using the think-pair-share strategy, the teacher invites students to '*say what you can see*' to a student sitting next to them, and then to the whole class. As shown in Table 1, this task is usually carried out over two 40-minute lessons.

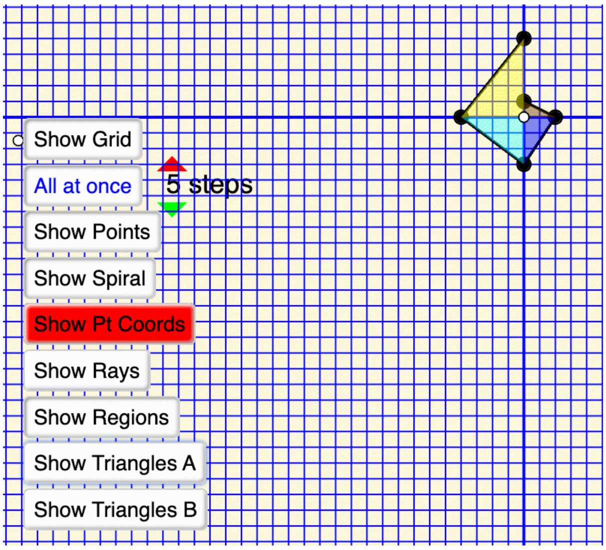
Table 1 – Lesson phases in using the spiral pattern task

Lesson phase		Classroom activity	Time (minutes)
Lesson 1	1. Whole-class introduction	Teacher presents the task.	5
	2. Individual work	Students construct the spiral pattern. Teacher, then, asks students to take note of any mathematics-related aspects that they can see within the pattern.	15
	3. Pair and share	Teacher invites students to discuss their ideas with the person next to them and then to share their ideas with the whole class. Teacher writes students' ideas on the board.	20
Lesson 2	4. Small-group work	Students select one idea from the board and investigate it further.	20
	5. Whole-class discussion and summary	Teacher asks groups of students to present their investigations and then summarises the main teaching points emerging from students' inquiries, making connections where possible.	20

The applet for this *spiral pattern* (see Figure 3), designed by John Mason, is intended to help students to alternatively use technology to construct the spiral. Drawing on the UDL principle of representation, the applet also offers opportunities to explore the spiral using a variety of options that include the use of the grid and gridlines, plotted points, coordinates of points, different sets of triangles and different regions. The applet, hence, provides an alternative way for students to access and explore the task, and opportunities for reasoning that stem from the possibilities that students can generate when using the applet.

See Appendix B for details of how to run the applet.

Figure 3 – Spiral pattern applet designed by John Mason



Designing Tasks for Mathematical Inquiry

Mathematical inquiry, for which students need to assume more responsibility for their learning (Calleja & Buhagiar, 2022), is not necessarily initiated with a real-life experience or phenomenon. In fact, a stimulus to inquiry can be a mathematical statement, prompt, or question (see, for example, the works of Blair, 2014; Foster, 2013; Skovsmose, 2001; Swan, 2005). In the *spiral pattern* task, the stimulus for inquiry emerges from the pattern of straight lines and can be used by the teacher to provoke students to ask questions about the resulting image, hence compelling students to investigate mathematics.

Learning mathematics through inquiry involves a process of sense-making, that is, students' ability to apply knowledge of a situation and connect it with existing knowledge. Within an inquiry approach to learning mathematics, the *spiral pattern* is intended to provide students with a variety of challenging experiences (Calleja, 2020) through which they can actively contribute ideas and construct mathematical meanings for themselves (Biccard, 2018). The tasks teachers select and the ways students negotiate mathematical meaning largely determine students' experiences of learning mathematics (Shimizu et al., 2010). Indeed, Doyle (1983, p. 161) points out that mathematical tasks, and the ensuing classroom activity, are "defined by the answers students are required to produce and the routes that can be used to obtain these answers". Hence, when using this task, it is important for teachers to consider carefully the cognitive demands.

Willis (2010) recommends that teachers should gauge tasks to an appropriate and productive level as gradual increase in the level of task difficulty, challenge, and structure is likely to facilitate student engagement. In a study with grade 5 and 6 students in Australia and China on making student participation in the mathematics class more inclusive, Barkatsas and Seah (2015) report that students tended to value tasks that were challenging but within reach. Challenge and accessibility are task design issues that informed the work of Swan (2005), Willis (2010), and Blair (2014). For example, Willis (2010, p. 17) recommends the use of tasks that offer an achievable-challenge. This means that tasks need to "require students to exert mental effort, performing a task that is just difficult enough to hold their interest but not so difficult that they give up in frustration". To promote challenge and collaboration, Blair (2014, p. 33) offers prompts (in the form of an equation, statement or diagram – see also www.inquirymaths.com for a range of examples), set just above students' current knowledge and intended to act as stimuli to provoke interest and curiosity. Swan (2005) developed a commended design framework for providing students with mathematical tasks that foster both conceptual development and problem solving. The suggested collaborative tasks involve students in evaluating mathematical statements, classifying mathematical objects, interpreting multiple representations, creating and solving problems, and analysing reasoning and solutions.

The Spiral Pattern as an Open Investigation

The spectrum of inquiry tasks is rather extensive and can be placed along a continuum ranging from teacher-directed to student-centred inquiry. Tafoya, Sunal and Knecht (1980) and Walker (2007) differentiated inquiry according to the type, task structure, and difficulty, and the degree of student engagement and responsibility. Similar to Tafoya et al. (1980), Staver and Bay (1987) classified inquiry lessons into *structured*, *guided*, and *open* inquiry. Within the *structured* type, which is predominantly teacher-directed, students are provided with the problem, the method, and the resources to solve it. For *guided* inquiry, which is characterised by teacher guidance, students are given the problem and the necessary resources, but it is then their task to find the appropriate strategies and methods to use. Within *open* inquiry, which is student-directed, students are tasked to decide about the problem, the methods, and the resources that they use. Drawing on this classification, Aulls and Shore (2008) propose that student and teacher roles and responsibilities also exist along a continuum. In *structured* inquiry, the teacher assumes responsibility for students' learning; in *guided* inquiry, the teacher shares responsibility with the students; and in *open* inquiry, responsibility is mostly on the students with the teacher acting as a consultant and facilitator.

The *spiral pattern* task assumes an *open* approach to investigating mathematics since *open* inquiry offers better opportunities for students' cognitive development as they take ownership of their investigations (Boaler, 2016; Swan, 2005). It is also designed as a non-routine investigation that heightens their interest, curiosity, and enthusiasm, thereby rendering the learning of mathematics more exciting (Stemm, 2008). The *spiral pattern* task presents a 'curiosity provoking situation' that should be stimulating for students and captivate their interest in investigating mathematics (Greenes, 1996). As they engage with the task, students need to make their own decisions, plan their own routes, choose methods, and apply their mathematical knowledge. When students engage with such work, they are involved in processes of exploration and explanation (Skovsmose, 2001). Students may focus strictly on solving the problem set or pose their own problems or questions for investigation. Their attempts and decisions to investigate can make mathematics more relevant and worthwhile for them. This encourages their creativity which may stimulate the development of more independent learning skills. The *spiral pattern* investigation, hence, takes a holistic approach to learning, connecting topics and, thereby, presenting a more complete view of mathematics.

The Spiral Pattern as a Low-Floor, High-Ceiling Task

An important feature of the *spiral pattern* task is that it provides multiple entry points for students to access the investigation. As a low-floor task, it is "accessible to a wide range of students" (Boaler, 2016, p. 84) since it is designed such that students can "access the mathematics inherent in the task at their current level of learning" (Russo et al., 2020, p. 49). One approach to make a task low-floor is to present the problem and ask students to describe what they see or how they see the problem. In such a scenario, the teacher would accept and write down (for example, by taking notes on the board) all ideas and contributions made by students. Furthermore, the teacher would encourage students to be creative and take risks in sharing their thinking and making explicit their observations. Drawing on students' ideas, the tasks would then offer multiple learning trajectories that can be personalised based on the interests and needs of each student.

The *spiral pattern* is considered as a high-ceiling investigation because it is intended to promote the curiosity of all students (Boaler, 2016) and, hence, the ideas generated by students about the spiral can be extended to high levels (either by students or with the support of the teacher). In other words, the *spiral pattern* provides access to higher order thinking (for example, instigating students to pose new problems) and explore more challenging mathematical content (Gadanidis, 2012). This means that, with support from the teacher, mathematical concepts, patterns and relationships may be extended to include more complex connections between ideas and more varied representations of ideas.

Supporting and Scaffolding Student Inquiry

To foster inquiry, the teacher acts as a challenger and an intervener; one who asks questions to encourage and stimulate student thinking and reasoning (Swan, 2005). Such teacher support provides agency so that students can determine the process and outcome of their learning. Hence, the guidance and support provided by the teacher is intended to help students to learn how to work more independently.

Mercer (1995, p. 48) describes this role as "the sensitive, supportive intervention of a teacher in the progress of a learner, who is actively involved in some specific task, but who is not quite able to manage the task alone". For this to happen, the teacher needs to be able to scaffold student learning through modelling and coaching (Hmelo-Silver et al., 2007) offering, in the process, space for students to think, share, discuss, make decisions, and come up with sound arguments and plausible solutions. Notably, to scaffold student learning, the teacher needs to draw student attention to significant ideas by providing “content knowledge on a just-in-time basis” (Hmelo-Silver et al., 2007, p. 100).

Just-in-time learning occurs when students focus their attention exclusively on the information that can be used (Calleja, Foster & Hodgen, 2021). Hence, students use specific knowledge for dealing with an encountered challenge. Students’ knowledge, at that particular moment, may be limiting the learning process. Encouraging and supporting students to engage in reflection is critical so that they can make decisions about what, how and when to learn. From a socio-constructivist viewpoint, the teacher needs to provide just-in-time scaffolding when learning outcomes are comprehensible to students but not easily achievable without support (Goos, 2004).

Design Features of the Spiral Pattern Investigation

Following a description of the *spiral pattern* task and its characteristics to promote an *open* approach to mathematical inquiry, I briefly discuss UDL and then explain how I draw on UDL principles to design the *spiral pattern* investigation so that it can make way for offering students more inclusive mathematics teaching. Then I discuss the three key design features and illustrate how investigating the *spiral pattern* offers both multiple entry points and a range of possibilities for all students to engage in independent and collaborative exploration of mathematics.

The Universal Design for Learning (UDL) as a Design Framework

A widespread approach for inclusive education is UDL which involves the use of pedagogy combined with technology aimed towards including all students in the learning process (King-Sears, 2009). According to Courey et al. (2013, p. 10), “UDL can be defined as a set of principles and techniques for use in the classroom along with the design of accessible instructional materials”. While some researchers tend to link UDL to technology-based interventions (for example, Rose & Meyer, 2002), others advocate a broader approach to inclusive education highlighting how teachers can design learning opportunities to engage all students (for example, Kortering, McClannon & Braziel, 2008).

In principle, through UDL teachers may design teaching so that all students can have better access to the whole curriculum, be challenged, find the challenge relevant and achievable, and eventually succeed (Hitchcock et al., 2002; Rose & Meyer, 2002). UDL applied to teaching mathematics offers teachers a framework that helps them focus on how best to support students to access mathematical content through different materials, technologies, and teaching strategies (Hunt & Andeasen, 2011; Thomas et al., 2015). The UDL framework is based on three key principles (see Table 2) related to providing students with multiple options of (a) representation, (b) action and expression, and (c) engagement (Kortering, McClannon & Braziel, 2008; Meyer & Rose, 2000). Representation, for example, involves making content accessible through multimedia such as images, videos, animations, text, diagrams and questions to help students interpret content. Action and expression refer to the way students communicate their learning. These may include pen and paper, an oral presentation, drawings, and representations. Engagement involves stimulating the interest and motivation of students to learn by participating in a variety of activities that include individual work, small group, and whole class discussions (Courey et al., 2013).

Table 2 – UDL principles

UDL Principle	Students
Representation	Have alternatives to work on task (e.g., pen and paper or animations with the use of technology)
Action and expression	Can communicate ideas in different forms (e.g., through text, drawings or oral presentation)
Engagement	Work individually, collaborate and communicate ideas (e.g., through small and whole group discussions)

By drawing on the above three principles of the UDL framework, I now discuss the three key design features (see Table 3) of the *spiral pattern* task and how it can be used by teachers to embrace diversity and create a more inclusive mathematics classroom setting.

Table 3 – Task design features

Feature	Tasks
1	Provide multiple entry points and encourage students to make contributions
2	Offer access to a broad and connected mathematics curriculum
3	Lead to struggles and achievable challenges through just-in-time support

Feature 1: Provide Multiple Entry Points and Encourage Students to Make Contributions

By exploring possible patterns within the resulting design, the teacher encourages students to generate their own observations and construct their own ideas about the *spiral pattern*. One effective way of doing this is to encourage students to '*say what you can see*'. In saying what they can see, students are encouraged to contribute an idea or an observation. Typical responses might be: I can see …

- Lines of increasing length
- Some lines that may be parallel
- The spiral generates right-angled triangles
- Lengths of hypotenuses follow a particular pattern
- Shapes which are almost trapezia
- A pattern within the sides making up the right-angled triangle
- Right-angled triangles with increasing areas
- Triangles which could be similar
- A pattern which never stops growing

At this early stage, shared observations and ideas are encouraged, not judged. Hence, there is no right or wrong response; each and every response merits consideration and can be an opportunity for exploration. Should other students have comments to make on a particular idea, they are encouraged to challenge and/or add further comments to it but also to think about related questions and come up with possible conjectures. For example, while someone might notice that the spiral generates right-angled triangles, another might add that the right-angled triangles have increasing areas. Students' inquiry of the areas of these triangles (see Figure 2) might lead them to discover that the answers lead to the sequence of triangular numbers. Here, the applet also provides for 'action and expression' (UDL Principle 2) so that students have an alternative way to access the task, work on it, and communicate their conclusions.

Feature 2: Offer Access to a Broad and Connected Mathematics Curriculum

Internationally, there has been emphasis on mathematics curricula to incorporate both the practical facet of mathematics and the more abstract one (Sullivan, 2015). Within both facets, students need to gain the ability to pose problems, make decisions, reason, solve problems, discuss mathematical meanings, and eventually communicate them.

Linked to the idea of a low-floor task offering multiple entry points, the *spiral pattern* task opens the possibility for students to access a range of topics within the mathematics curriculum, including:

- Areas of right-angled triangles
- Triangular numbers including formula
- Lengths of lines and the use of Pythagoras' theorem
- Using circle theorem of angle on diameter
- Areas of circles (using lengths of lines as diameters)
- Expressing the n^{th} term of a sequence
- Expressing the n^{th} sum of a sequence
- Limits of a sequence – of lengths, areas, and slopes
- Conditions for lines being parallel
- Angles in parallel lines
- Using trigonometric functions for finding angles
- Slopes and equations of lines

These topics are mostly linked to the mathematical content strands of algebra, geometry, shape, space and measure. Calleja (2020) provides more details about how the *spiral pattern* task encourages mathematical explorations, calculations, the writing of mathematical solutions, and possible connections between topics that students might be able to make.

Feature 3: Lead to Struggles and Achievable Challenges Through Just-In-Time Support

The *spiral pattern* investigation design follows the three criteria identified by Mason and Johnston-Wilder (2006, p. 7) so that students have:

- relevant experiences from which to extract, abstract and generalise principles, methods, perspectives and ways of working with mathematics;
- stimuli appropriate to the concepts to be worked on;
- a supportive and compatible social environment in which to work.

Hence, the lesson structure provided below offers teachers ways of presenting mathematical content and concepts through a stimulating task but, more importantly, it offers students diverse ways of working through the task. This task can be planned over a four-phase lesson structure which includes: (1) exploring the spiral; (2) gathering preliminary ideas; (3) testing conjectures; and (4) sharing mathematical meanings.

Following the '*say what you can see*' phase and the list of ideas contributed, students are invited to select one idea and explore it further—first individually and then within a small group. Such an approach is intended to open up mathematics and help students to think in divergent ways, looking beyond the starting situation while incorporating and connecting different areas of mathematics. Diversity is critical because the variation among students makes it possible for novel ideas to emerge (Askew, 2015). This process is also likely to lead to initial struggles and would, at this stage, require students to pose new problems, questions and conjectures, to explore, and eventually make an attempt to answer them.

In opening up a task to allow for multiple ideas, content and conjectures to be investigated, teachers need to be confident and believe in the mathematical competencies of their students. Hence, they need to make sure that all students are present, engaged, have opportunities to participate, are confident that their contributions matter and, as a result, feel a sense of belonging to the classroom community (Carrington & MacArthur, 2012). In this scenario, students can take risks as they venture into the unknown trusting the benefits of their inquiry, collaborative and independent work. The teacher, on the other hand, plays a key role in convincing students to pursue their own ideas and inquiry and that, irrespective of the outcome, such inquiries would eventually be worth pursuing. To do this, and help students pursue with their thinking, the teacher can use prompts and questions (see Table 4). The proposed questions serve both as probes to support and challenge students' ideas as well as prompts to extend their thinking.

Table 4 – Purposeful questions and prompts

Lesson Phase	Questions and prompts
Exploring the spiral Students observe the spiral pattern and then say what they can see	• *What does the pattern consist of?* • *What does the spiral generate?* • *What mathematical ideas come to mind when you see this spiral?* • *Which mathematical topics could be related to this spiral?*
Gathering preliminary ideas Students share preliminary ideas to the whole-class	• *How did you get this?* • *Where have you seen something like this before?* • *What is changing in this spiral?* • *How is it changing?* • *What else do you see and might be worth exploring?*
Testing conjectures Students test conjectures while working in small groups	• *Is the spiral pattern going anywhere?* • *Can you form any hypotheses?* • *Can you think of any counter examples?* • *Why did you change your strategy?* • *Can you suggest a different strategy?* • *What is a sensible way to record this data?* • *What conclusions can you draw from this data?* • *How can we check that calculation without doing it all again?* • *What patterns can you see in your data?* • *What reasons might there be for such a pattern?* • *Convince me with a sound argument?* • *Can you predict the next one?* • *Can you come up with a formula to generalise your finding?*
Sharing mathematical meanings Students present their mathematical meanings and provide justifications to the whole-class	• *What method did you use?* • *What other methods have you considered? Why?* • *How did you make the link?* • *What challenges did you encounter?* • *How did you solve the challenges encountered?* • *What do you conclude from this?* • *What other question/s would you pose at this stage?*

Aspects of mathematical modelling—that include looking at one aspect of the spiral, listing possible assumptions, making conjectures, assigning variables and formulating the problem in terms of a set of algebraic expressions, looking out for patterns, using mathematical techniques to solve the problem, testing conjectures, and generalising—will help support students in the process of their exploration. The teacher can facilitate this process by asking students to explain their thoughts about the problem, to state what is known and unknown, and to provide ways of solving it.

A Task-based Curriculum

In this paper, designing tasks for inclusion is conceptualised as offering a challenging situation (or problem) within which all students "can achieve something worth-while" (Burghes, 1984, p. 51) because they see it as meaningful, accessible, and have the readiness for it. Here, the supportive role of the teacher is key to help students pursue their work on the task. The teacher must be knowledgeable enough to open up, challenge students' mathematical thinking and ideas, and scaffold learning. One example of how the teacher can do this is through prompts and questioning (refer to Table 4 for some examples) that stimulate deeper thinking during students' inquiry process. This approach to an inquiry curriculum is inclusive because it offers students opportunities to choose the content of their learning and experience both the joys and frustrations of mathematical exploration and inquiry (Biccard, 2018). Hence, at the design stage, tasks need to offer choice for students so that they can provide them with possibilities to select the ideas, and the methods and representations of their ideas.

Within an inclusive classroom that embraces diversity, students do not necessarily learn mathematics at the same time or have to make progress at the same pace. Diversity is a key feature to engage students in deep learning and, in inquiry teaching, it is valued and sought for (Askew 2015). Also, in task design drawing on a UDL framework, student diversity is embraced by principles of representation, action and expression, and engagement. Hence, in designing for inclusion, I explain how task designers may address issues of unequal access and opportunities so that all students can fully participate within a broader mathematics curriculum. I make an argument for task-based teaching which is an approach through which teachers, rather than presenting a topic, use tasks that are accessible, extendable and encourage problem-posing, problem-solving, thinking, questioning and creativity (Swan, 2005). The challenge, as I see it, is not in conceptualising such tasks, but in engineering and refining them so that teachers may be able to use them. This is indeed a challenge for teachers, and their students too, particularly in contexts and countries where, like Malta, ability grouping, a compartmentalised curriculum and an over-emphasis on success in examinations is still prevalent. Local research (Calleja & Buhagiar, 2022) suggests that when teachers offer students a more participatory and task-based approach to learning mathematics, some students, particularly those for whom transmission teaching has helped them succeed in their examinations, tend to resist it. In what follows, I provide suggestions for curriculum and task designers, and also teachers about how they can provide more inclusive learning opportunities for their students.

In providing suggestions for task-based teaching, I use task design principles and link these with inclusion, UDL and inquiry teaching. Mathematical tasks need to include:

1. a *purpose*, that is, a justification that aids students' understanding that a particular task requires specific ways of working which eventually lead all students to learn;
2. a *context* which, drawing on Skovsmose (2001) can have three references to reality: (a) reference to pure mathematics (involving rich mathematical content as in the spiral pattern task), (b) reference to a semi-reality (a reality constructed by the task designer and that exists outside of the classroom, like a best buy problem at a supermarket), or (c) reference to a reality where the task is enacted in a real-life situation;
3. a *process* that involves students in meaning making by supporting them to 'assert' and justify their ideas rather than to 'accept' what the teacher says or suggests (Mason & Johnston-Wilder, 2006);
4. a *product* that offers multiple forms of representation and which may not necessarily be presented as finished and, hence, may require further thinking and questioning; and
5. a *curriculum scheme* that emphasises the interconnected nature of mathematics (Swan, 2005) and within which tasks, cross-referenced to learning objectives, would fit (see Figure 4 for an example).

Figure 4 – Example of a task-based curriculum scheme

Scheme of Work	1st Term – 14 Week Planner			Year 7
Solving Problems and Number Card Games	**Estimating and Measuring Quantities**	**Sorting Decimal and Fraction Puzzles**	**Exploring Rectangles**	**Planning a Survey**
A range of tasks dealing with number work and including powers of 10 cards, problems related to discovering hidden numbers and a snakes and ladders game. During these tasks students ideally work in pairs. (Estimated weeks – 2)	Situated in a lab, four stations are set up with tasks focusing on length, time, mass and volume. Students work in groups to first estimate given quantities and take correct measurements with the appropriate apparatus. (Estimated weeks – 1)	The puzzles involve students in sorting and in discovering relationships between fractions and decimals. Moreover, these activities involve students, working in small-groups, in problem solving. (Estimated weeks – 2)	On 1 cm^2 squared paper, ask students to draw rectangles with areas of 7, 9, 12 and 24. Encourage students to investigate perimeter; factors; multiples; prime & square numbers; HCF and LCM. (Estimated weeks – 2)	Students working in small-groups are asked to decide about a topic for investigation. The survey involves compiling a questionnaire, analysing the data gathered and presenting the results to the whole-class. (Estimated weeks – 1)
• Add and subtract natural numbers up to 1000 • Read and write whole numbers in figures and words • Powers of 10 • Multiply and divide natural numbers by powers of 10 • Multiply and divide natural numbers by a single digit • Multiply natural numbers by a two-digit number using the partitioning method and the standard written method • Divide natural numbers by a two-digit number using the repeated subtraction method	• Convert metric units of length, mass and volume to smaller units and vice-versa • Add, subtract, multiply & divide quantities of length, mass and volume and solve problems • Use different units of time; determine time intervals in hours and minutes; write time using the 12-hour & 24-hour clock & convert 12-hour to 24-hour clock & vice-versa; read & use a timetable and a calendar • Read scales in real-life	• Arrange decimals in ascending & descending order • Write equivalent fractions and change fractions to decimals (fractions with denominators being factors of 100) • Reduce fractions to their lowest terms; find fraction of quantity • Add/subtract two fractions with same/different denominator; multiply a fraction • Change fractions & percentages to decimals & vice-versa; find the percentage of a quantity • Add & subtract decimals	• Perimeter of simple shapes • Find the area of simple shapes by adding unit squares; units of area: mm^2, cm^2 & m^2 • Factors & multiples; even & odd numbers; find common multiples of two numbers • Recognise prime numbers and write numbers as a product of their prime factors • Squares and square roots, use the calculator to find squares and square roots	• Collect data using observation, surveys and experiments; compile and interpret frequency tables for un/grouped discrete data; draw and interpret bar charts • Compute, manually and using a spread-sheet, the mean, mode, median and range for ungrouped data
Classifying Triangles	**Investigating Offers and Best Buys**	**Discovering Polygon Angle Facts**	**Thinking about Parallel Lines**	**Drawing a Tennis Court Plan**
Students are provided with two worksheets, scissors and glue. They are required to cut out the given set of triangles on one worksheet and to classify them in a two-way table provided in the other worksheet. (Estimated weeks – 1)	Situated in a lab, stations are set up with tasks dealing with offers and best buys. Students work in groups to sort out the best deals and offers justifying their reasoning through calculations and/or written explanations. (Estimated weeks – 1)	The tasks presented involve students in finding out the sum of the interior angles of triangles and quadrilaterals. Students will have the opportunity to discover relationships, find connections and create definitions. (Estimated weeks – 1)	Students are given a worksheet showing a pair of parallel lines and a transversal line passing through the two lines. With a protractor, they investigate the resulting angles and comment about their results. (Estimated weeks – 1½)	Students are asked to do a scale drawing of the school's tennis court. While on site, students draw a sketch of the court, decide which dimensions to take and use their measuring tape to accurately measure distances. (Estimated weeks – 1½)
• Distinguish between different types of angles • Estimate and measure angles • Angle sum property in triangle • Classify triangles (equilateral, scalene, isosceles and right-angled)	• Multiply and divide decimals by an integer • Related problems involving money	• Use a protractor to measure and draw angles up to 180° • Angle sum in quadrilateral from triangle property • Problems involving angles in a triangle and in a quadrilateral	• Parallel lines, alternate, corresponding & interior angles • Problems with alternate, corresponding & interior angles • Problems involvingangles on a straight line, angles at a point and vertically opposite angles	• Use the ratio notation to compare two or more quantities; write ratios in simplest form • Use a scale when it is written as a fraction or as a ratio • Interpret the scales on plans and draw simple scale drawings

A task-based curriculum scheme rejects the notion that mathematics is presented as a sequence of isolated, disconnected, and sequential topics. Instead, it draws attention to the connectivity among topics. I focus on teachers, and particularly those who are new but committed to providing a more inclusive inquiry approach through task-based teaching. Designing for inclusion through inquiry teaching is a long-term developmental process that requires time—both for teachers to conceptualise and put into practice and for students who need to make sense of their more active roles and what teachers expect of them (Calleja & Buhagiar, 2022; Calleja, Foster & Hodgen, 2023). Indeed, for education systems like Malta, that rely on prescribed and content loaded syllabi, ability grouping and ongoing testing, the shift to teaching for inclusion through inquiry is anything but easy. Towards this end, any suggestion for teachers must go hand-in-hand with a robust professional development (PD) programme that offers co-learning experiences for teachers (Goodchild, Fuglestad & Jaworski, 2013). Ideally, teachers are not receivers of knowledge from PD providers who then expect them to implement suggested practices in their classrooms. On the contrary, teachers need to take a central role in design, implementation, evaluation, and development. They need to be treated as experts of teaching and shoulder responsibility for learning as they become co-constructors and co-learners alongside curriculum and task designers. In PD, all participants—teachers, PD providers, curriculum, and task designers—act as central agents that feed each other as they share insights and actions and develop knowledge of and about teaching. My proposal, also based on research with teachers in Malta, is for teachers to take risks but also to apply inquiry according to how they think it would work for them and their students in practice (Calleja & Buhagiar, 2022; Calleja, Foster & Hodgen, 2023). For example, Calleja, Foster, and Hodgen (2023) suggest that, in implementing inquiry, teachers in Malta make modifications to the proposed four-phase lesson structure to address contextual constraints arising from limited teaching time, their perceived needs of students, the support required by students to complete a task and the prescribed content to teach. Teachers may adopt a more scaffolded approach to teaching by deliberately providing students with support—in the form of questions, prompts, discussion and telling—at different phases during the lesson, particularly when they think that their students would need it and can make use of it. A scaffolded approach could, for instance, involve the breaking down of tasks into a sequence of smaller tasks. This, I think, is an important insight for curriculum and task designers. To be applicable to different educational contexts, tasks need to be designed in such a way that they offer flexibility and adaptability for teachers by incorporating: (1) scaffolding strategies targeted to individual needs of students; and (2) a sequence of smaller tasks that eventually lead students to work towards the main task. Hence, in designing for inclusion tasks need to offer possibilities for teachers' supportive interventions in such a way that they "help students to engage with tasks that would otherwise be too demanding" (Calleja, Foster & Hodgen, 2023).

This paper set out to illustrate how task features, for the design of the *spiral pattern* task, can be linked to UDL principles of representation, action and expression, and engagement. An important feature is that students need to gain access to learning through multiple means. Hence, tasks need to offer students different ways to access mathematical knowledge, to actively engage with it and to communicate their mathematical meanings effectively. In the *spiral pattern* task, using the applet and adopting the '*say what can you see*' strategy (combined with teacher support through a think-pair-share activity) are intended to embrace and support diversity within students. Through a scaffolded approach to teaching, students can have opportunities to make sense of the information given, select their preferred ideas for inquiry, act on it, make their own contributions, learn collaboratively and express themselves freely.

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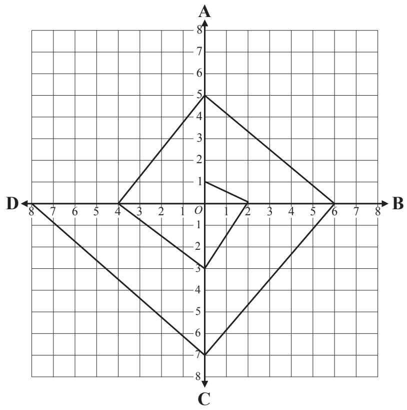
Appendix A – Iterations of the spiral pattern task

Within an inquiry approach to teaching mathematics, the design and development of the *spiral pattern* task involved a number of iterations. Each iteration included a number of trials in secondary school classrooms that eventually led to variations and improvements.

Iteration 1

The first iteration used numbered 'axes' marked A,B,C,D (see Figure 5): students constructed the spiral by mapping points A1 to B2, B2 to C3, C3 to D4, D4 to A5 and so on. Students were then encouraged to colour the spiral and work out a number of closed questions (for example, finding a rule to explain how the spiral is growing). This task was initially intended to stimulate the curiosity and interest of 11 to 13-year-old students who struggled with and feared to fail in their attempts at doing mathematics.

Figure 5 – Graph from first iteration



Iteration 2

This involved replacing the labelling of the axes using Cartesian coordinates as shown in Figure 2. The plotting of points now changed to (0,1), (2, 0), (0, −3), (−4, 0), (0, 5) and so on. This system offered more opportunities for investigation linking the resulting pattern with more than one aspect of the mathematics curriculum. This task, implemented with 11 to 13-year olds, encouraged students to identify the pattern through which the spiral was growing.

Iteration 3

Following multiple trials with groups of different abilities and levels, it was evident that different students could come up with different mathematical observations. This led to opening up the task and presenting it as an open investigation. The idea was to move from a problem-solving task (asking students to identify the pattern of increasing triangles and work out a solution and formula for it) to developing it as a problem-posing situation where students could make observations, ask and answer their own questions. The initial prompt, hence, changed from asking a question to prompting students to notice, make observations, ask questions and explore mathematics within the task. Numerous trails indicated that this more open version of the task supported students to make connections among different topics (see Feature 2 in section 3).

Appendix B – Running the Spiral Applet

The applet for the spiral pattern (see Figure 3) was designed by John Mason using the free *Cinderella* interactive geometry software. It can be run in your web browser, without installing *Cinderella*, at:
https://educationaldesigner.org/ed/volume5/issue17/article68/applets/spiral.html

Alternatively, if you have *Cinderella* (available from https://cinderella.de/) installed, you can download the file
'spiral.cdy' from:
https://educationaldesigner.org/ed/volume5/issue17/article68/applets/spiral.cdy

Note that, after starting the applet, you will have to click on the red arrow to set the number of steps and click one of the 'Show...' buttons to see anything.

About the Author

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