Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Use of Mathematics (Pilot) USE3

Mathematical Comprehension

Thursday 27 May 2010 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 45.

Advice
- You are advised to spend 1 hour on Section A and 30 minutes on Section B.

Section A

Answer **all** questions in the spaces provided.

Use **Half full or half empty?** *on the Data Sheet.*

1 (a) For the conical cocktail glass referred to in the article, with maximum cross-sectional radius of 4.6 cm and depth 5.5 cm, show, using the formula for the volume of a cone, that the maximum volume of liquid, V cm^3, that it can hold is approximately 122 cm^3. *(2 marks)*

(b) (i) Show clear calculations to confirm that, to fill this glass so that it is half full, it should be filled to a depth of approximately 4.37 cm. *(4 marks)*

(ii) Hence show that the depths of the liquids in the two glasses in **Figure 1** on the Data Sheet are in the ratio 1.26:1. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

2 Draw a diagram to show why the radius, x cm, of a cylinder y cm from the origin, for the conical glass shown, is given by $y = \frac{5.5}{4.6}x$. *(2 marks)*

QUESTION PART REFERENCE

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3 (a) For the conical glass, the volume, $V\,\text{cm}^3$, varies with the depth of liquid in the glass, $y\,\text{cm}$, according to $V = 0.233\pi y^3$.

Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}y}$ and evaluate this when $y = 1$ and $y = 4$. *(4 marks)*

(b) Interpret your answers to part **(a)** in terms of the glasses in **Figure 1** on the Data Sheet. *(2 marks)*

QUESTION PART REFERENCE

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4 (a) In the article, 11 cylinders were used to calculate an estimate of the volume of the conical glass.

In this case, calculate the error in the estimated volume as a percentage of the actual volume, 121.87 cm^3. *(2 marks)*

(b) If, instead, 22 cylinders are used in a similar way, a more accurate value for the volume can be found.

In this case, find the volume of the largest cylinder. *(3 marks)*

(c) Explain why, even when using a greater number of cylinders, this method will always give an overestimate of the volume of the cylinder. *(2 marks)*

QUESTION PART REFERENCE

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5 Show, using integration, that the volume of liquid, $V\,\text{cm}^3$, in the glass from the article, whose surface outline is modelled using the function $y = 0.35x^2$, when filled to a depth of $d\,\text{cm}$ is given by $V = 4.49d^2$. *(5 marks)*

QUESTION PART REFERENCE

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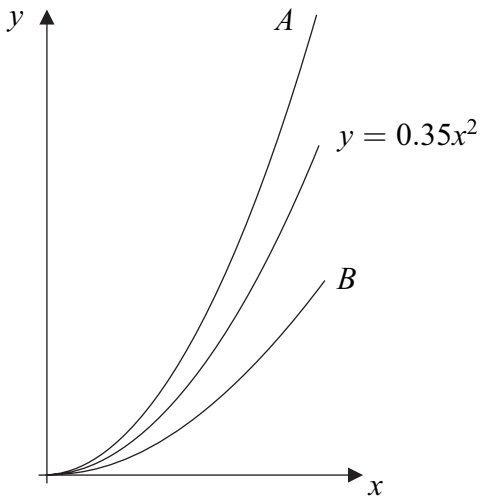
6 The graph below shows a sketch of the outline of the wine glass from the article, which was modelled using the function $y = 0.35x^2$, together with the outlines, A and B, of two other glasses.

Suggest possible quadratic functions for these two different glasses.



(2 marks)

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Section B

Read carefully the article below and answer **all** questions in the spaces provided.

Daylight hours

You will no doubt have noticed that the number of daylight hours varies throughout the year. On any given day, the number of hours of daylight you experience depends on where you are on the Earth's surface. For example, in our winter, if you are at the North Pole you will have no daylight hours, whereas if you are there during our summer you will have no hours of darkness. On the other hand, at the equator you will have about 12 hours of daylight throughout the year. Therefore, in the middle of June in the northern hemisphere, you will have somewhere between 12 and 24 hours of daylight.

Figure 9 *Measuring angle of latitude*

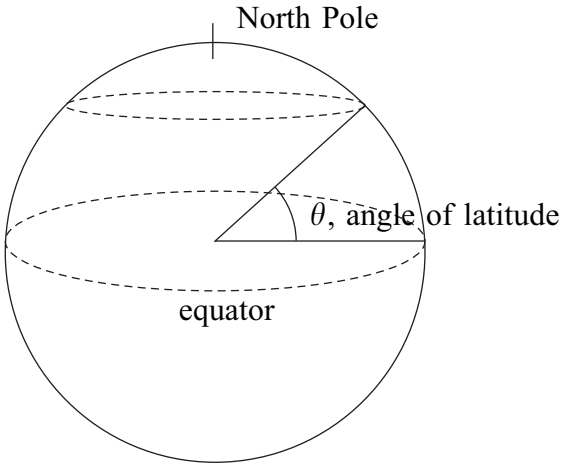
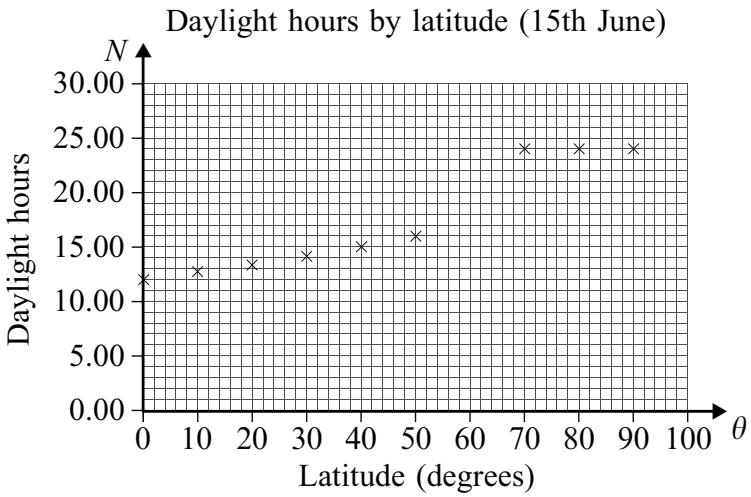


Figure 10 *Latitudes of some major cities in the northern hemisphere*

Latitude, degrees	City	Country
60	Helsinki	Finland
56	Edinburgh	Scotland
51	London	England
40	Madrid	Spain
36	Tokyo	Japan
22	Hong Kong	China
19	Mumbai	India
7	Lagos	Nigeria
0	Kampala	Uganda

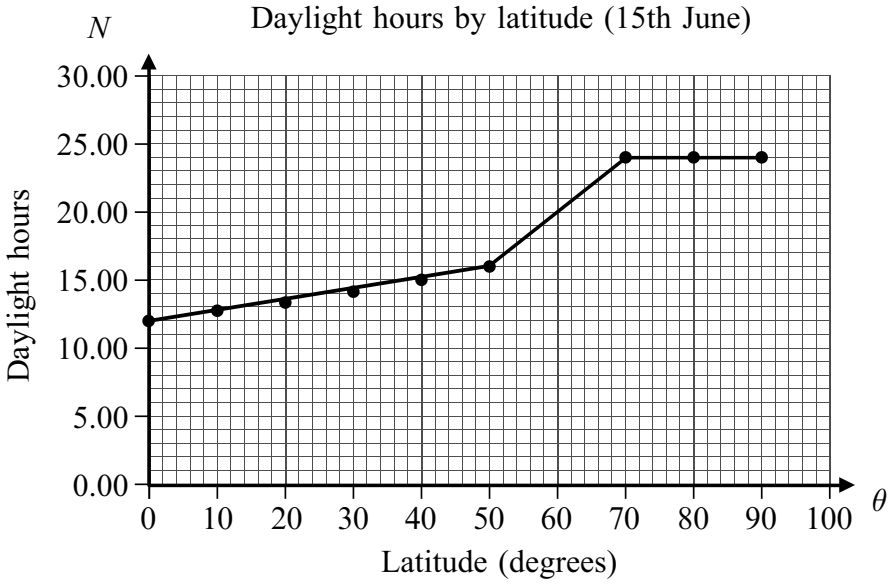
Figure 9 shows how the angle of latitude, giving a measure of how far North you are, is measured from the centre of the Earth. The angles of latitude for some major cities in the northern hemisphere are given in **Figure 10**.

Figure 11 *Graph showing how daylight hours, N, vary with angle of latitude, θ °, in mid-June*



The graph in **Figure 11** shows how the hours of daylight vary with latitude on the 15th June. The formula used to calculate the values shown in this graph is very complex so three straight lines have been used to model the graph of the data in **Figure 12**.

Figure 12 *Graph showing daylight hours, N, modelled by three straight lines*



7 (a) Find equations for the straight lines used to model how the number of hours of daylight, N, in the northern hemisphere in mid-June varies with angle of latitude, θ°, for:

(i) $0 \leqslant \theta \leqslant 50$; *(3 marks)*

(ii) $50 \leqslant \theta \leqslant 70$. *(3 marks)*

(b) Use the appropriate model that you found in part **(a)** to give an estimate of the number of daylight hours on a mid-June day in Helsinki. *(2 marks)*

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8 The function $N = 12 - 3\cos d^\circ$ can be used to model how the number of hours of daylight, N, varies with the number of days, d, into the year ($d = 1$ on 1st January) in Madrid.

By considering this model, state:

(a) the maximum number of daylight hours there are during a year in Madrid;

(b) the minimum number of daylight hours there are during a year in Madrid;

(c) the period of the function. *(3 marks)*

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9 By using the function $N = 12 - 3\cos d°$, find how many days into the year it is when there are first 14 daylight hours in Madrid.

(4 marks)

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END OF QUESTIONS

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