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The Development of a Workshop for Cultivating Leaders of Mathematics-Grounding Activities in Class

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Abstract

Teacher professional development (TPD) is an important topic for school teaching and learning, but the challenging part is how to design quality and sustainable TPD that results in sound teachers' learning. This research paper presents a cascade model of TPD used by the Just Do Math project and designed to nurture any number of teachers to teach mathematics in a more student-centered approach, rather than teaching mathematics in an inflexible way. The one-day workshop for cultivating leaders of mathematics-grounding activities in class (MGA-in-class), discussed in this article, is built on our previous TPD cascade model of scaling up the cultivation of designers, leaders, and teachers of mathematics-grounding activities for camps (MGA-for-camp). This paper focuses on presenting the process of developing and evaluating the one-day workshop for leaders of MGA-in-class. The development includes the processes of (1) how to transform activities designed for use in special camps to activities for use in regular classes, (2) how to cultivate the designers of MGA-in-class to be the facilitators in the one-day workshop for leaders of MGA-in-class, and (3) how to organize and evaluate the one-day workshop. Though the evidence shows that the one-day workshop has not recruited as many participants as previous TPD, the collected evidence provides useful information for future adjustment.

1. Introduction

Teacher professional development (TPD) has influential impacts on teacher learning (Borko, 2004; Gibson & Brooks, 2012) and is viewed as the key to educational reform (Lieberman & Pointer Mace, 2008). In previous studies on TPD the focus has been on its various forms (Borko, 2004; Haßler, Bennett & Damani, 2020), but the trends of studying TPD have transferred from personal to the learning community (e.g. DuFour, 2004), from an individual perspective to a situational perspective of learning (e.g. Putnam & Borko, 2000), from one-shot learning to continuing learning (e.g. Webster-Wright, 2009), and from workshops to online courses (Haßler et al., 2020). Curriculum reform is one key factor that motivates and influences TPD (Hoekstra & Korthagen, 2011; Little, 1993; Scribner, 1999). Although there are many studies tackling the issues of TPD related to curriculum reforms, we know little of how a TPD program supports teachers to notice the essence of the reformed curriculum and to prepare appropriate lessons from both theoretical and practical perspectives.

JDM aims to integrate three fundamental elements of mathematics teaching and learning:

1. products designed by teachers that can motivate students' mathematics learning,
2. various types of professional development workshops,
3. the context in which students learn with the designed products (see Lin & Chang, 2017).

JDM is a multi-year project that began in 2013 and has developed in three stages: *mode* stage (2013–2016), *economy* stage (2017–2019), and *power* stage (from 2020). The three stages of describing the JDM project are analogized from Bruner's (1966, p. 44) three ways of structuring the domain of knowledge, including the modes of representations in which it is put, its economy, and its effective power. Each stage undertakes all three fundamental elements above but the missions and goals differ (see Table 1). The mode stage was more or less the construction and refinement of the functioning of the JDM project in an organized mode. The economy stage was the transformative stage that brought the JDM project closer to the educational field. The power stage is now being organized. In the economy and power stages, the missions of previous stage(s) continue for scaling up. It is planned to integrate mathematical content, contexts, and various skills emphasized on PISA 2021 mathematics framework, such as critical thinking, creativity, research and inquiry, self-direction, etc. All three stages deal with scaling up teachers and students' involvement in the JDM project. After briefly introducing the *mode* stage, this article discusses the development and evaluation of a one-day workshop for cultivating leaders that is part of the work of the *economy* stage.

Table 1 The missions of the mode and economy stages of the JDM project

	Three fundamental elements of mathematics teaching and learning		
	Designing Products	**Offering Workshops**	**Learning Context**
Mode stage (2013–2016)	Mathematics-grounding activities used in camps (MGA-for-camp)	• Series of half-day workshops for designers of MGA-for-camp • Series of one-day workshops for (1) leaders of MGA-for-camp, and (2) teachers of MGA-for-camp	Fun-Math Camps (in schools on weekend or in vacation)
Economy stage (2017–2019)	• Mathematics-grounding activities used in regular classes (MGA-in-class). • MGA-in-class Instructional videos	• Series of half-day workshops for designers of MGA-in-class • Series of one-day workshops for (1) leaders of MGA-in-class and (2) teachers of MGA-in-class	Lessons in schools

The JDM project started in the mode stage with Fun-Math Camps based around mathematics-grounding activities (MGA-for-camp). The camps aim at providing opportunities for students to learn mathematics with fun, use their own mathematical sense, and to think mathematically via dialogues among peers (Lin & Chang, 2019; Lin, Wang & Yang, 2018). Schools apply to host Fun Math camps, and all fees are covered by the Ministry of Education, Taiwan. The applicants submit a plan including details of how many classes will be offered (usually 15–30 students per class), choice of weekend, winter vacation, or summer vacation, and whether they need JDM support teachers to run the camp. Each class uses two to three MGAs-for-camp and is led by certificated MGA-for-camp teachers over one half day.

The JDM project implemented a cascade professional development model with three types of workshops: workshops for cultivating designers of MGA-for-camp; workshops for cultivating teachers to learn to implement MGA-for-camp with students, and workshops for cultivating leaders to lead the teachers' workshops. (Lin, Yang & Wang, 2016). To date it has certificated 296 designers, 497 leaders, and 23,361 teachers with respect to MGA-for-camp. The certificated teachers of MGA-for-camp are eligible to attend workshops for cultivating leaders of MGA-for-camp.

In 2016 more and more teachers of the MGA-for-camp, particularly for elementary students, showed their practical concerns of applying MGA-for-camp in regular school teaching. They were motivated by two reasons. One is the curriculum reform in Taiwan, transforming the *ability-based* curriculum (mainly knowledge and skills) to a *core competence-oriented* curriculum (a combination of knowledge, skills, and attitudes and an emphasis on nurturing life-long learners), for advancing school teaching and learning in all subject matters and cross-disciplinary activities. This reform was proposed in 2014 and implemented in 2019 (Ministry of Education, 2020). The other reason is that teachers had experienced how students' active thinking is advanced through participating in various MGA-for-camp (Lin et al., 2018).

The mathematics teacher educator-researchers (MTE-Rs) of the JDM project therefore deliberated upon the practicability of transforming MGAs-for-camp to mathematics-grounding activities that could be used in regular classes (MGA-in-class). They set up the end goal of the economy stage to be cultivating as many mathematics teachers as possible to practice competence-oriented instruction using MGA-in-class. To accomplish the goal, the following issues needed to be tackled:

1. designing an initial set of MGAs-in-class,
2. creating workshops for cultivating designers, leaders of the TPD, and then teachers of MGA-in-class,
3. disseminating MGA-in-class to schools.

In this article, we focus on the development and evaluation of the one-day workshop for cultivating the leaders of MGA-in-class; those who will lead the workshops to cultivate the teachers of MGA-in-class. The decision to complete the workshop in one day was made because the teachers invited to participate had already been leaders of MGA-for-camp. Therefore, they all had *experience* of leading other teachers in how to implement MGAs-for-camp, and their facilitation of students' active engagement in mathematical thinking when implementing MGA-for-camp had already been evaluated. Moreover, their learning in the workshop was controlled in one day to achieve the *basic understanding* of MGA-in-class. Accordingly, we chose a one-day workshop for them to work with given set of MGAs-in-class. The one-day format was efficient, enabling MGAs-in-class to be widely implemented as soon as possible.

2. Theoretical Background

In this section, we review the literature on the approaches, features, and motivations for professional development as the theoretical background and later present the considerations taken into account for developing the one-day workshop.

2.1 Analyzing Video to Enhance Learning from Professional Development

Recent studies have commonly discussed using a lesson study, video clubs, or animation to develop teachers' *noticing* students' prior knowledge or thinking for enhancing in-the-moment decision making (e.g. Amador & Carter, 2018; Chazan & Herbst, 2012; Coenders & Verhoef, 2019; Lewis, Perry & Hurd, 2009; van Es, 2011). Jacobs, Lamb & Philipp (2010) observed that teachers' noticing students' thinking involves three interrelated skills: (a) *attending* to students' actions (called strategies in the original), (b) *interpreting* students' thinking (understanding in the original), and (c) *deciding* how to respond to students' thinking (understanding in the original). In view of the fact that noticing is one critical component of teachers' professional expertise (Sherin, Jacobs & Philipp, 2011), we reflect on how it can be a tool for cultivating both the designers (including for their role as workshop facilitators) and the leaders of MGA-in-class.

The one-day workshop was planned to adopt the practice of video-based analysis in the TPD to notice students' thinking and actions regarding the given instruction of one MGA-in-class. The designers of MGA-in-class, were invited from previous designers of MGA-for-camp. After designing quality MGA-in-class with MTERs' theoretical support, they continued to make its video in 15 minutes to connect theories and practices, and then they became facilitators for the one-day workshop, helping participants analyze the video and co-prepare mathematics lessons.

In order to help teachers effectively learn during their professional development, Santagata and Angelici (2010) found that certain prompts help preservice teachers to reflect productively when analyzing videos of mathematics lessons. The prompts focus on "(a) the learning opportunities provided by the lesson activities, (b) student learning from the activities as it could be observed in the video, (c) specific students' difficulties, and (d) possible alternative strategies the teacher could have implemented and their potential impact on student learning" (p. 342). These prompts will be employed when the designers of MGA-in-class make videos of MGA-in-class, and will help the later activity of analyzing videos in the one-day workshop to be focused and work smoothly.

2.2 Features of Quality Professional Development

Many features are considered in empirical studies of effectiveness of TPD intervention (Darling-Hammond & McLaughlin, 1995; Desimone, 2009; Garet et al., 2001; Hawley & Valli, 2000; Knapp, 2003; Putnam & Borko, 1997, 2000; Wilson & Berne, 1999). Borko, Jacobs and Koellner (2010) categorized TPD features into two groups: content characteristics and process and structure characteristics. There are two key content characteristics of high-quality TPD: *situated in practice* and *focused on students' learning*.

For instance, the mathematical content of MGA-in-class aims to correspond with school mathematics (situated in practice) and to engage students in mathematical thinking (focused on students' learning). These two features can help teachers learn alternative instructional decisions and actions through working on the interactions between MGA-in-class and students' ideas - that is, how students think and learn.

Concerning the process and structure characteristics of high-quality TPD, the five critical features of TPD workshops include:

1. the processes of modeling *instructional strategies* for teachers,
2. engaging teachers in *active learning*,
3. building a professional learning community for teachers,
4. *school based* learning activities and
5. ongoing and sustainable activities over time.

These five features will be included in the self-evaluation criteria of the one-day workshop to investigate participants' satisfaction.

2.3 Teachers' Motivation for Professional Development

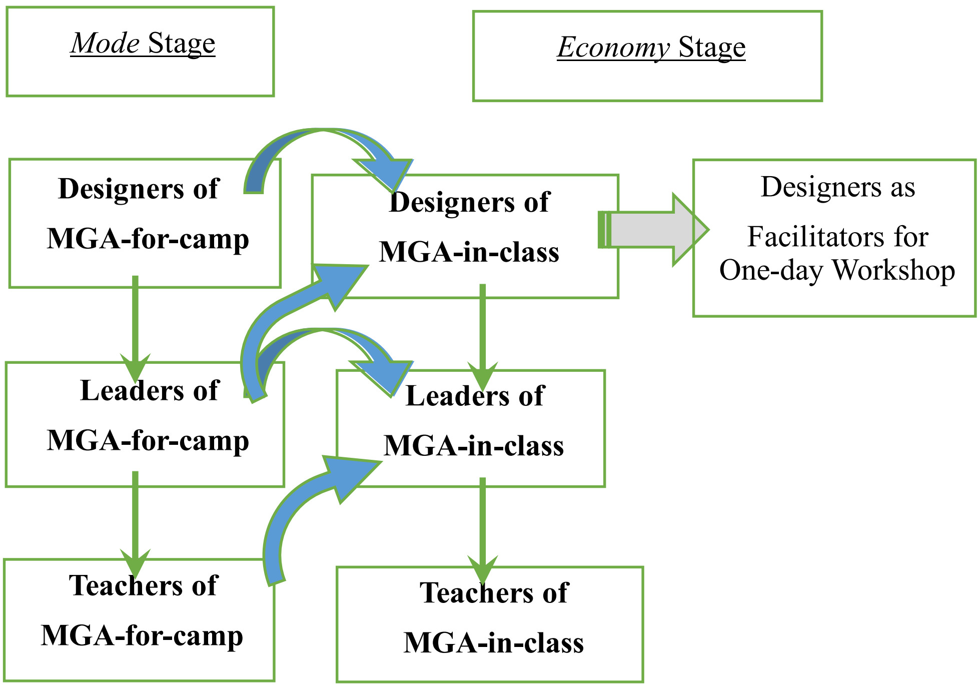
Effective professional development involves not only the model itself, but also the individual teachers' actual learning or change that takes place during their professional development. Adults' learning is closely connected to their *motivation* (see Zepeda, 2012). Therefore, motivating the participant teachers' perspectives on the workshop can help to make their learning more effective. Keller's (1987) ARCS model identifies four major conditions - attention (A), relevance (R), confidence (C), and satisfaction (S) - that have to be met for learners to become and remain motivated. This model is thus worth applying to evaluate an individual's motivation for learning as one index of effective professional development. The four ARCS conditions are reference points for the organization and evaluation of the one-day workshop. Hence the evaluation of the one-day workshop will include participants' self-evaluation of their *attention* to the content of the workshop, the *relevance* of the content to their own mathematics teaching, their *confidence* in practicing the content, and their *satisfaction* with the workshop.

3. The Development of the One-day Workshop

3.1 Overall plan

The *economy* stage of the JDM project is set to speed up the generation of teachers who can bring the three elements of having fun, making sense of mathematics, and thinking mathematically in their mathematics teaching, whilst most importantly keeping the quality of the *mode* stage. The evolution of TPD in the JDM project from the *mode* stage to the *economy* stage is briefly shown in Figure 1. Within the stage, the cultivation follows the cascade model - that is, the designers (in their new role as workshop facilitators) lead candidate leaders, and qualified leaders lead teachers, to learn different sets of MGAs-for-camp/MGAs-in-class in workshops. Between the stages, the designers of MGA-in-class come from those having experiences of designing MGA-for-camp or receiving several certifications of leaders of MGA-for-camp; the leaders of MGA-in-class come from those having experiences of leading MGA-for-camp or receiving several certifications of teachers of MGA-for-camp; while there is no restriction between stages for the teachers of MGA-in-class. In addition, leaders and teachers of MGA-for-camp/MGA-in-class will receive the certifications based on different sets of MGAs-for-camp/MGAs-in-class. Usually one set of MGAs-for-camp/MGAs-in-class includes 3 to 4 modules of MGA-for-camp/MGA-in-class.

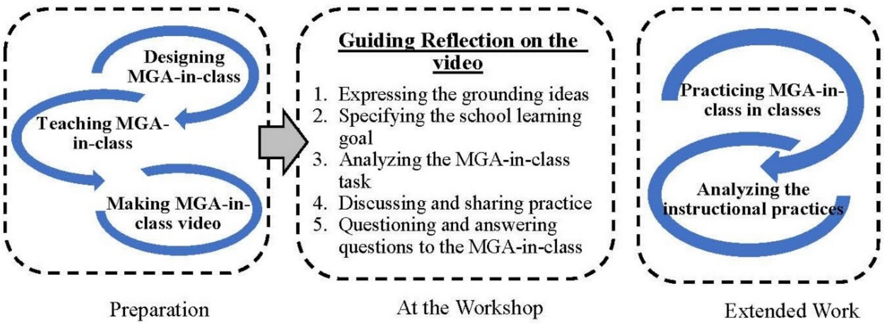
Figure 1 – The evolution of TPD from the mode stage to the economy stage



As shown in Figure 1, experienced designers, who were also certificated as leaders of MGA-for-camp, were recruited to participate in a series of workshops to design MGA-in-class. Figure 2 shows that they began their work by designing MGA-in-class, conducting a teaching experiment, and making an instructional video of MGA-in-class (see the preparation phase of Figure 2) which they analyzed in the one-day workshop (see the workshop phase of Figure 2).

We will elaborate upon the transformation of MGA-for-camp to MGA-in-class in section 3.2, approaches to cultivating the designers of MGA-in-class to be the facilitators in the one-day workshop in section 3.3, and the arrangement and evaluation of the one-day workshop in section 3.4.

Figure 2 – Actions before, during, and after the one-day workshop



3.2 Transforming MGA-for-camp to MGA-in-class

As mentioned above, the educational environment in Taiwan is undergoing curriculum reform, requiring a change in school mathematics teaching and learning. Although the experimental MGAs-for-camp were also designed to promote students' mathematical competencies, they are not easy to apply in regular school classes, because they take a long time for students exploring mathematical phenomena before entering formal rules. In addition, motivating students is not as easy in regular classes. Therefore, it is necessary to transform the MGAs-for-camp into tasks suitable for school mathematics classes. To make a smooth transition from the present MGAs-for-camp to MGAs-in-class, one efficient way is to transform the existing pool of well-designed MGAs-for-camp into new tasks that can be implemented in regular classes and come closer to reaching the goals of the new mathematics competence-oriented curriculum. These goals include learning mathematics from daily life contexts and natural language, experiencing mathematics from observing the patterns, nurturing mathematical disposition, providing opportunities to learn mathematics through sense-making, and learning how to use tools correctly (Ministry of Education, 2018).

When designing the MGAs-in-class, the designers were asked to analyze the big ideas of the original MGAs-for-camp and to identify the corresponding topic and grade level in the school curriculum. Moreover, the designers needed to compare the traditional instruction of this topic to the instruction of the MGAs-for-camp as well as to integrate the three rationales and five principles proposed by the MTE-Rs into their design of the MGAs-in-class. These three *rationales* are

1. mathematics learning requires attention to both students' cognition and emotion,
2. making sense of mathematics is the process of grounding mathematical learning, and
3. mathematics learning happens mainly in the co-construction among students.

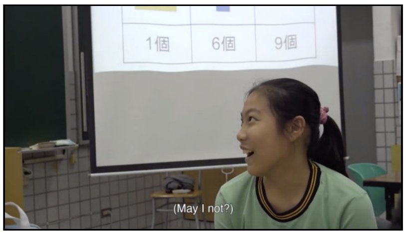
The five *principles* are

1. animating student mathematical thinking,
2. constructing students' mathematical sense,
3. co-constructing mathematics among students and between students and the teacher,
4. teaching with diagnostic intervention, and
5. permeating the corresponding topic content of the mathematics textbooks into the MGAs-in-class.

The first three principles are adopted from three rationales to be concise for designers to catch. The differences between them from MGA-for-camp to MGA-in-class are (1) designing MGAs-for-camp emphasizes more on students' emotional engagement than MGAs-in-class, (2) designing MGAs-for-camp pays more attention on physical manipulation while MGAs-in-class more on mental images, and (3) designing MGAs-for-camp stresses more on students' interaction while MGAs-in-class more on the interaction between students and teacher. The latter two principles are especially given for linking school mathematics instruction. The fourth principle is meant to foresee students' various errors and provide instructional intervention at proper timing. The fifth is to connect the MGA-in-class to the corresponding mathematical topic of school mathematics.

We take the MGA-in-class, *Carpet Factory* as an example to elaborate how it was transformed from an MGA-for-camp into an MGA-in-class by considering its underlying rationales and principles.
The 15-minute instructional video edited from two continuous classes can be found at https://reurl.cc/ZQ9nAl.

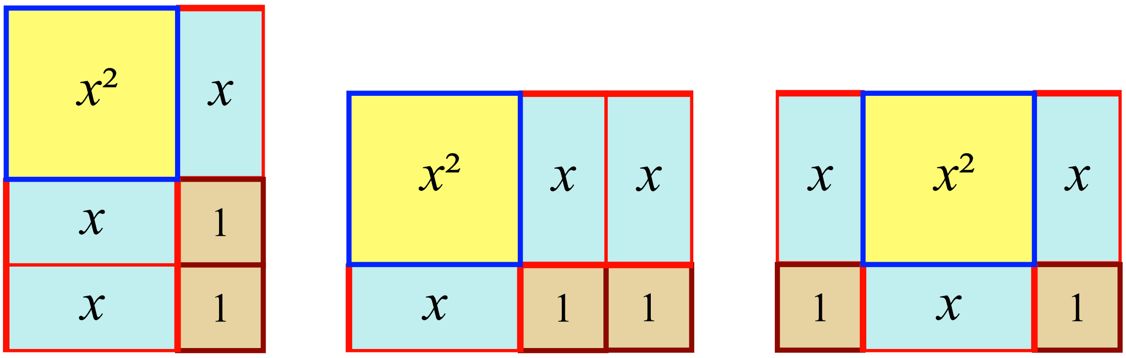
Video of Carpet Factory Lesson



https://reurl.cc/ZQ9nAl

The task was originally designed as an MGA-for-camp, called *Squaring the Squares and Rectangles* (Deng, 2014). The goal was to learn about factorizing quadratic polynomials by manipulating geometric shapes of squares and rectangles. Students are given a kit of square cards with fixed side length is labelled x, square cards with side length 1, and rectangular cards with side lengths of x and 1. In the MGA-for-camp, students first get acquainted with the cards, their side lengths and areas first based on their prior knowledge of area formulae. The students then use these three types of cards to construct various areas given in algebra and to explore the quantitative relationships between the cards and the constructed shapes. One example is to make a rectangle with given area of $x^2 + 3x + 2$ (i.e. one square card with area of x^2, three rectangular cards with area of $x \cdot 1$, and two square cards with area of $1 \cdot 1$). Commonly seen shapes constructed by students can be found in Figure 3. Students can observe their side lengths of the rectangles and discuss the relationship between the given area $x^2 + 3x + 2$ and the rectangular area calculated from its side lengths: $(x + 1)(x + 2)$. To complete the MGA-in-camp, students have a competition with their peers based on the given kit, including the playing cards with the given areas to construct and worksheets to fill.

Figure 3 – Geometric shapes of the given area constructed using a set of cards



Although factorization, the topic of this activity, belongs to the domain of algebra, manipulating geometric shapes is included to help students link abstract algebraic polynomials to concrete results of geometric shapes. It provides an intuitive and genetic approach (Klein, 2016) for students to actively think about the polynomial itself. Moreover, the game playing and manipulation process of the activity originate from the students' natural language (e.g. the area sum of the six cards equals the area of the big rectangle) rather than generalized and structured mathematical language or objects (e.g. the addition of the three terms of x^2, 3, and 2 equals the product of the two expressions $(x + 1)$ and $(x + 2)$. Such a process provides an opportunity for students to think and reason mathematically. However, this inquiry process of the original MGA-for-camp, involving preparation, development, and play, is very time-consuming, and difficult to implement in a regular lesson of only 40–50 minutes.

Therefore, in the MGA-in-class the square with area of x^2 is fixed at the corner of the large rectangle to shorten students' exploration time. The revised version efficiently guides students to focus on the connection between the *numbers* of rectangles (e.g. 3) and small squares (e.g. 2) and the two *side lengths* of the large compound rectangle (e.g. $x + 1$, $x + 2$). Additionally, the teachers' guide accompanying the structural designed worksheet suggests that the first quadratic polynomials presented have a prime constant term (only one combination of side lengths) with composite constant terms later. This arrangement greatly shortens students' free exploration. After experiencing several tasks with different numbers of rectangles and squares, the ideas of factorization gradually emerge. Bringing such activity into mathematics learning in classes makes it easier for students to think about how the factorization of their textbook works.

In this MGA-in-class, the exploration activity is changed from the original MGA-for-camp construction of various geometric shapes to a cultural issue. The daily life context of a carpet factory is provided to motivate students to think about how cutting and reassembling the regular rectangular carpet works. This not only shortens the game playing, but also ignites students' curiosity and increases fun to find the solution.

During the academic year of 2017–2018, the MTE-Rs led groups of designers. The resulting MGA-in-class followed the MGA-for-camp designs and integrated the key competencies of mathematics into various mathematical topics. By early 2019, there were 38 MGAs-in-class covering different mathematics topics, mostly adapted from the resources of the MGAs-for-camp.

3.3 Cultivating Designers of MGA-in-class to be Facilitators in the One-day Workshop

We now present what the designers of MGA-in-class needed to do to become facilitators of the one-day workshop, how they did it, and which capabilities they are expected to develop. Four actions with the support of the MTE-Rs were required to be taken. The first was to design the MGA-in-class (e.g. Carpet Factory) adapting them from the MGA-for-camp (e.g. Squaring the Squares and Rectangles). The second was to try out the design of the MGA-in-class in classes to evaluate its effectiveness at integrating it with students' learning of school mathematics. The first and second actions were cyclical for revising the design of MGA-in-class and supported by an MTE-R. The designed products of MGA-in-class were evaluated by the review committee of MTE-Rs, and good ones were approved for the subsequent actions. The third action was to implement and videotape a lesson of MGA-in-class in around 2 continual classes (about 80 to 90 minutes) with the support of an MTE-R. The fourth action was to construct a 15-minute video by editing the 90-minute videos with the support of the review committee of MTE-Rs and one expert media-maker. The last two actions were taken with the cooperation of a photo studio provided by the JDM project. It is noted that they implemented and recorded the MGA-in-class with two different classes in order to make good exemplary MGA-in-class videos. The major concern was that all staff, especially the photo studio team, could become familiar with the flow of the instruction in order to collect useful footage for leaders' later selection.

The five design principles of MGA-in-class, described in section 3.2, could not be provided as strategies to design but were referenced as criteria for evaluating the MGA-in-class and particularly for editing its video. The review committee of MTE-Rs targeted the first two principles on students' physical/mental manipulation and connections of students' ideas to develop mathematical ideas, targeted the third principle on students' interactions with others and their communication of mathematical ideas, targeted the fourth principle on teachers' questioning to evaluate students' misunderstanding and understanding, and targeted the fifth principle on the correspondence of the content between MGA-in-class and the mathematics textbooks. These concerns echo the principles of the reformed mathematics curriculum, including to experience mathematics starting from natural language rather than abstract mathematical objects, from experiencing mathematics as a practical science of patterns as well as nurturing students' disposition, and to learn mathematics through sense-making and using tools. The designs of MGA-in-class provide the learning opportunities for students to make sense of the subject, to use tools through manipulation and communication among others with mathematical language evolved from their initial expressions, and to discover mathematical relationships with their dynamic disposition with which develop to an appreciation of mathematics as humanistic activities.

Through designing and revising the MGA-in-class videos, these designers could deepen their understanding of the principles of designing the MGAs-in-class and their connections to the reformed mathematics curriculum. Such a deep understanding provides a foundation for the participants to discuss how MGAs-in-class can promote students' mathematical competencies under the first prompt suggested by Santagata and Angelici (2010): the learning opportunities provided by the lesson activities. By implementing their MGAs-in-class, they could experience students' various responses, decide how to deal with them, and engage in the instructional interactions. This experience then can become the resources for being aware of students' learning from the MGAs-in-class, noticing students' possible difficulties, and taking feasible teaching strategies. These resources are beneficial at facilitating a discussion of the other three prompts suggested by Santagata and Angelici (2010): student learning from the activities, specific student difficulties, and possible alternative strategies the teacher could have implemented and their potential impact on student learning.

During the process of videotaping and editing the video, the designers had to identify critical events to represent how students' mathematics competence can be developed through actively engaging in MGA-in-class and connected to the rationales of the reformed mathematics curriculum. Through editing the MGA-in-class videos with the MTE-Rs' support, the designers intensively attended to students' actions, explicitly interpreted their critical thinking, and reflectively selected proper teaching strategies to develop their mathematics competence - that is, we adopted editing MGA-in-class videos as an approach to advancing designers' noticing, as defined by Jacobs et al. (2010), expecting that they can facilitate leaders of MGA-in-class to discuss critical moments and their underlying principles based on their noticing experiences. The experience of being a designer, screenwriter, teacher, and editor of the MGA-in-class video with MTE-Rs who can link theories and practices effectively is crucial for designers of MGA-in-class to develop their competence in facilitating leaders of MGA-in-class.

4. The Arrangement and Evaluation of the One-day Workshop

The one-day workshop was developed according to the conception of the four-element TPD model, including goals, contexts, theories, and structures (Sztajn, Campbell & Yoon, 2009). To respond to the demands of teachers regarding how to implement competence-oriented mathematics instruction and the training of a pool of qualified teachers in a limited time period, the *goals* of the one-day workshop were to:

1. advance the participants' understanding of the competence-oriented mathematics curriculum,
2. enhance the participants' competencies of implementing MGAs-in-class in their mathematics classes or adapting them to their students,
3. provide participants with experiences of co-planning instruction of some textbook topic where MGA-in-class can be integrated, and

4. cultivate participants as potential facilitators for disseminating MGAs-in-class and leading mathematics lesson co-preparation workshops in Taiwan.

The *contexts* of developing the one-day workshop include teachers facing the new educational challenge of practicing a competence-oriented curriculum and how MTE-Rs can help to construct a workable TPD learning model for mathematics teachers to quickly learn the essence of competence-oriented mathematics in a short period of time. These two key contextual factors impel the one-day workshop underlying the previous practices of MGAs-for-camp and their application in the later co-preparation workshop.

The *theories* involved in developing the one-day workshop, as discussed previously in Section 2, include several considerations for ensuring quality professional development. They are (1) employing videos (e.g. Amador & Carter, 2018) to enhance teachers' noticing (Jacobs, Lamb & Philipp, 2010) of students' learning within the learning community (e.g. Coenders & Verhoef, 2019), (2) controlling quality of the content, process, and structure of the one-day workshop (Borko et al., 2010), and (3) perceiving teachers' motivation to take part in the workshop and their learning via the ARCS model (Keller, 1987). For example, after learning in the one-day workshop, teachers reflected that the workshop helped them to inspect what they knew about a competence-oriented curriculum and how MGA-in-class can be integrated in regular classes (attention/relevance). They also reflected how the design of MGA-in-class can really help them to comprehend and proceed with a student-centered mathematics lesson, and the learning community helped them to learn from peers (satisfaction). Though most of them were not confident in their teaching with MGA-in-class or a competence-oriented mathematics curriculum, they were more confident in integrating MGA-in-class into their instruction.

In February 2019 the JDM team proposed and organized the pioneering one-day workshop, including the structure and selection of its content based on the organizational experiences of the MGAs-for-camp and the aforementioned considerations for MGAs-in-class. The course *structure* of the workshops (see Table 2) comprised one *lecture* session delivering the goals, procedures, and the essence of the workshop, the *corpus* session of learning activities with three parallel lines for participants interested in junior high school mathematics, high-grade (i.e. grades 5–6) elementary school mathematics, and middle-grade (i.e. grades 3–4) elementary school mathematics, and a *whole group discussion* reflecting on and communicating about the whole day's learning as the closing session. In each parallel line of the corpus session, the implementation phase (see Figure 2) included three co-preparation activities of different mathematics topics - that is, three MGAs-in-class.

When organizing the one-day workshop, the initial MTE-R of the JDM project arranged a *preparation meeting* to gather all designers of MGA-in-camp, MTE-Es, and MTE-Rs to deliberate on the goals, the flow of the learning activity, and the key elements of the workshop in order to control its quality before it started. The core competence-oriented mathematics curriculum with the corresponding tasks and instructional videos of the MGAs-in-class were the focal content, and the preparation for teaching the specific mathematics topic with the corresponding MGA-in-class was the main mission of the participants in the workshop. Several rounds of the one-day workshop were set as the preliminary trial of leading the TPD at the economy stage. One designer of MGAs-in-class was set as the facilitator in the workshop along with the guidance of one MTE-E or one MTE-R to run the workshops.

Table 2 – Structure of the one-day workshop

1	*Lecture* (Introduction)	50 min
2	*Three parallel lines* of video-based learning activity (each including 5 leading stages on 3 MGA-in-class tasks and instruction; 90 minutes of co-preparing each one)	270 min for each parallel line
3	*Whole Group Discussion* (Reflection and Communication)	30 min

Each co-preparation activity in the parallel lines session (see Table 2) was conducted in five *leading stages* that guide reflection on instruction (e.g. Calandra et al., 2008; Collins et al., 2004; Rich & Hannafin, 2008) of each video-based learning activity. These leading stages echo Santagata and Angelici's (2010) prompts for driving teachers' reflection when engaging in productive video-based mathematics teaching. They include the following.

1. Manipulation: to play the MGA-in-class task (as a student) with peer teachers.
2. Communication: what mathematics can be learned with the played MGA-in-class.
3. Exploration: connecting the relationships between the played MGA-in-class and the specific mathematical topics in different grades.
4. Pedagogical connections: expressing the key learning points, sharing students' learning difficulties, and the required competencies (including prior knowledge) for the specific mathematics topic.
5. Discussion and conclusion: the timing of applying the MGA-in-class in mathematics lessons, and the instructional strategies of linking mathematical content and the MGA-in-class.

Regarding the learning sources, the mathematical topic content of the MGAs-in-class and the instructional videos were incorporated into the learning activities of co-preparing for teaching. Blomberg and colleagues (2013) proposed five research-based heuristics of using videos in pre-service teacher education as follows:

1. identifying learning goals,
2. choosing the instructional approach,
3. choosing video material,
4. addressing limitations, and
5. aligning assessment with instruction/goals (Blomberg et al., 2013).

Their proposed third heuristic of choosing video material differs from the current study. The greatest difference in this study is that the mathematical content of the video was not only developed by the video leader, but the video was also edited by the leader. Moreover, the video leader also used these video materials to instruct other peer teachers in the one-day workshop. This video leader is the designer of the MGA-in-class as well as the facilitator of the one-day workshop. The consideration of letting designers be the facilitators reflected the previous mode stage of the JDM project, whereby peer teachers could easily communicate with each other. In this way, the communication among peer teachers and the theories and practices of the design can be linked easier compared to up-to-down TPD.

The main difference in using video in TPD between this study and others is that most studies focused on the effects of video usage (e.g. Gaudin & Chaliès, 2015; Tekkumru-Kisa & Stein, 2017), but not on how these videos were constructed and edited by teachers, and their professional knowledge and skills were developed through this process. The instructional videos of MGA-in-class were constructed by the designers, including their previous preparation of designing the content of the MGA-in-class tasks, the lesson preparation and pre-shooting of the instruction, the shooting of the formal instructional video, as well as the follow-up work of editing the video into 15 minutes. The construction, lesson preparation, instruction, and production of the 15-minute videos was not only individual work, but also community work involving MTE-Rs and sometimes peer teachers who were involved in implementing the instruction. During this process, MTE-Rs provided theoretical knowledge to guide the designers on noticing how students' mathematical competence was developed by MGA-in-class, which involved students' interactions, teacher's critical questioning, and feedback. The designers were asked to reflect on how they integrated the actions of playing games, manipulation, and communication through their making and editing of instructional videos and their interaction with students by connecting the theoretical knowledge. They then gathered these reflections for their later instructional work in the one-day workshop as a facilitator. Moreover, such reflections are key sources for MTE-Rs to inspect their professional development and students' learning from MGA-in-class, including their motivation, attitudes, and mathematical thinking.

The one-day workshop provided rich opportunities for participants to co-prepare three MGAs-in-class through community discussion (collective participation) on the instructional videos of MGA-in-class (video-based learning) with the support of the MGA-in-class developers. A set of one-day workshops helps start to cultivate the leaders of MGA-in-class who will in the future lead mathematics lesson co-preparation for teachers using MGA-in-class in the following workshops (see Figure 1).

In order to sustain the participants' learning in the one-day workshop, they were asked to complete two missions after the workshop (see the phase of extended work of Figure 2). The first was to select and implement an MGA-in-class, and the second was to analyze their own teaching and students' learning. The JDM team then evaluated the participants' reports and selected those participants who could notice the strengths and weaknesses of their MGA-in-class teaching in regular classes. They then became the MGA-in-class leaders who can lead the lesson co-preparation workshops for cultivating teachers of MGA-in-class.

5. Conclusions and Reflections

Regarding the development of the one-day workshop, we have some reflections on the individual work already implemented and still to be implemented. The first is the task transformation from MGAs-for-camp into MGAs-in-class. In order to keep the advantages of game playing and manipulation to motivate students' active mathematical thinking in the MGAs-for-camp, we think the culture-relevant elements, such as the Carpet Factory, that are close to the students' world can be included in the task transformation to shorten the exploration time while still igniting their mathematical curiosity and interest in thinking during regular classes. Moreover, the task transformation needs to keep the essence of the MGAs-for-camp, specifically (1) linking the big ideas of the topics (the importance and appropriateness of the content), (2) using a variety of mathematical objects, (3) ensuring the fitness and fun of the game playing design, and (4) involving the reification of the mathematical learning objects. Especially, in our informal investigation of why some teachers could implement MGA-in-class in their classrooms, we found that their strategy is to integrate the content of MGA-in-class into the contents of textbook in their instruction. It means that teachers restructured the textbook content when using MGA-in-class. Considering providing support for teacher autonomy (Kennedy, 2005), the following modification of JDM project include providing suggestions for teachers how to integrate both contents in mathematics teaching and shortening the instructional time of MGA-in-class from two classes to one class (40-45 minutes).

Second, during the cultivation of the designers to become the facilitators of the one-day workshop, we found that these experienced designers had strong practical experience of analyzing the curricular materials and of teaching. However, especially when making the 15-minute instructional videos, they easily put themselves in the field situation to notice students' actual reactions, but did not consider these reactions from an analytical perspective in regards to mathematical competence. To tackle this challenge, we arranged different types of collaborative groups to broker support for the designers, including peer teachers, MTE-Es, and MTE-Rs, and found that it helped with TPD (see Chang et al., 2020), especially in terms of connecting theories and practices.

Third, the TPD of one-day workshops was expected (1) to accumulate the participants' understanding of students' learning characteristics, power, and interests connected with the essence of the competence-oriented mathematics curriculum from watching and discussing various instructional videos through the designers' support, and (2) to empower leaders' capabilities in leading mathematics lesson co-preparation with the designed products of MGA-in-class. The main difference between such mathematics lesson co-preparation from a lesson study (e.g. Fernandez & Yoshida, 2004) is that teachers do not need to collaboratively design the lesson and instead see the lesson in an actual class for the follow-up steps of studying the lesson. These two steps are provided for participants by the one-day workshop as the materials for co-preparing the lesson. Our reflection here is that although the TPD of one-day workshops can provide the convenience of collecting various teachers from different schools in one community to co-prepare lessons, it is not easy for them to receive immediate feedback from the community when they go back to their classes for practice. To solve this problem, we have provided further support of connecting and sustaining teachers' communication through social media.

Fourth, according to our tracking data, only some participants were able to maintain their professional development learning after the one-day workshop. Some participants did not intend to be leaders of MGA-in-class, but rather aimed to improve their own classroom teaching, while others had low confidence in being good leaders. Accordingly, we will pay attention to enhancing teachers' commitment to being leaders and improving teachers' confidence in being leaders.

The lesson co-preparation workshop is the follow-up of the one-day workshop and is currently being prepared. The lesson co-preparation workshop will also consider the features of quality TPD learning (e.g. Desimone, 2009) and integrate the five critical features - that is, the workshop will involve

1. modeling instructional strategies to the corresponding MGAs-in-class and mathematical lessons,
2. the participants' engagement in active learning,
3. the construction of a professional learning community among the MGA-in-class-teacher facilitators and MGA-in-class teachers,
4. a type of school-based TPD with lesson co-preparation workshops, and
5. learning activities within the workshops that are ongoing and sustainable.

After the first trial of the one-day workshop and reflecting on the feedback from the participants, it was found that the difference between the key learning goals and missions of the workshops between the mode stage and the economy stage might not be explicit enough for the participants, which means they might not understand how to achieve the competence-oriented instruction through integrating MGAs-in-class in textbooks used in their regular instruction. To make this difference explicit, in the power stage the two activities of *formative assessment* and *diagnostic instruction* based on the products of MGAs-in-class will be integrated into upcoming one-day workshops to lead teachers to learn how to make good use of their knowledge with these two activities in their instruction with the specific MGAs-in-class. The course was then adjusted from *three* MGA-in-class learning activities to *two* in one day. In this way, the participants can better concentrate on their learning, deepen their understanding of MGA-in-class, and discuss how to integrate it in textbooks used in their regular instruction.

We conclude with the contributions of this study. The impact of this one-day workshop model is promising when the participants are well prepared and selected. In order to prepare the designers as facilitators, professional activities requiring their meta-practice (e.g. their involvement in the design cycle and video editing) are recommended. The products of the meta-practice can then be adopted as the learning materials (e.g. the designed products of MGA-in-class modules and their instructional videos) in the one-day workshop. The participants need to have relevant experience of the learning materials; i.e. implementation of the designs of MGA-for-camp. Additionally, the MTE-Rs and participants' interactions (i.e. the form of collaborative community) are integrated in running the one-day-workshop model. MTE-Rs can refer to the learning materials to engage participants to make sense of and to have connections with the rationales, principles, theories, and practices. Participants can provide their practical suggestions. This connection may enhance participant designers' and MTE-Rs' understanding and generalization of what and how different MGA-in-class modules can develop students' mathematical competence.

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