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Numerical Methods – A Curriculum Mystery

Charles Lovitt

Abstract

This opinion paper explores and questions why numerical (iterative) methods, so widely used in business and scientific endeavours, are largely underrepresented in school mathematics curricula. Mathematics topics such as solving quadratic equations by using the quadratic formula or using algebraic procedures to solve problems in measurement, geometry or probability are deeply entrenched in the teaching strategies employed by teachers and in the resources developed to support those behaviours. Iterative approaches to the same topics offer an alternative, and in many instances more efficient, approach. They are often more suited to struggling students and can lead to greater success and deeper understanding of the underlying mathematical concepts. Classroom examples are offered, the potential advantages discussed, and finally some thoughts are given on why in this digital STEM age numerical approaches have not penetrated school curricula to the extent they could or should.

Overview

Am I so out of touch with prevailing realities, conventions, and purposes of maths programs? Or are we all so trapped in some methods, procedures, and traditions long past their use-by-date that they are nigh on impossible to be re-evaluated?

We build curriculum to provide students with all the tools they need for their mathematical journeys. Some years ago, I studied a university course titled *'Numerical methods that (usually) work'* based on Acton (1970). It was about the power of empirical approaches to solving problems as an alternative – or additional – tool to go alongside formulaic approaches. I was entranced by the power of some of these approaches, so suited to this digital age, so suited to a wide range of student abilities, and I became convinced that within a few years they would feature prominently in school mathematics courses at all levels. That this has not happened is, and remains, a mystery.

In this article, which I offer as a highly personal commentary, I outline some samples of the approaches, and then discuss some various general advantageous features. Finally, I ponder why numerical methods have failed to penetrate school curriculum in any substantive way.

Professional Development Context

In my various roles supporting teachers to build or enhance their personal repertoires of teaching and learning skills, I have become a lesson collector - but I collect for a reason. The lessons I collect I argue are ‘interesting’. By this I mean professionally worthy of serious discussion as they illustrate how various current interest areas within maths education are being or might be interpreted at the classroom level. Their purpose therefore is as practical vehicles to generate professional learning discussions at the school level around various current issues and interest areas within school maths programs.

The lessons that I find, collect and codify have titles such as Multo, The Architect’s Puzzle, Snakes and Ladders, Heads and Legs, Maths in Motion, Algebra Walk, Licorice Factory, and Fraction Estimation. They all currently stored within the Australian Association of Mathematics Teachers Maths300 project (AAMT, n.d.). They are available to professional development leaders anywhere to plug them into their networks to generate discussions about notions of problem solving and reasoning, meaningful contexts, genuine understandings, effective use of technology, equity and differentiation, pedagogical strategies, assessment techniques, explicit teaching, learning theories, interconnectedness, and the ever present balancing act between skill development and open-ended investigative approaches. In other words, all the big ideas that can feed into teacher design of classroom lessons. By and large I feel moderately satisfied that the professional discussions have been productive as teachers constantly seek to expand the quality of their personal teaching repertoire and to develop rich and balanced programs.

When I reflect on lessons that have had an impact and those that haven’t, there is one particular lesson type that stands out. It disturbs me. It has not had the impact I think it deserves, and as mentioned above, that mystifies me as to why. The university course I studied and subsequently explored in classrooms used iterative empirical approaches very different from traditional formal analytic approaches. I was impressed and convinced that these methods could and should become commonplace and replace much of traditional formula methods. Indeed, these approaches are very evident in mathematical modelling in all sorts of business and scientific contexts such as climate change, economic theories, weather forecasting, population projections and town planning. Yet school mathematics structures have largely ignored this reality.

To illustrate this lesson type, I offer the two classroom lessons below. The major idea they both highlight and promote is an empirical iterative approach to problem solving. I hope the reader will indulge my argument by way of personal experiences.

Lesson 1. Area of a Circle

Some many years ago, I was appointed to a suburban high school. The mathematics classes were so called ‘ability streamed’. In Grade 8 there were 7 groups: 8A, 8B …. through to 8G. I was the last maths teacher appointed to the school that year – no prize for guessing which group awaited me. 8G already had sadly acquired a “We are no good at Maths” culture; commitment and confidence not being their forte. The classes followed a common course and sat a common exam. At one stage of the course a classic piece of Grade 8 content appeared. And I knew the question below was to be on ‘Friday’s Test’. What was I expected to teach? Well, we had presumably drilled solving algebraic equations for just this event.

Problem

The area of a circle is 40 cm^2. Find the radius.
Calculate your answer to 2 decimal places.

Solution

$\pi R^2 = 40$

$R^2 = 40 / \pi$

$R = \pm\sqrt{40 / \pi}$

$R = \sqrt{40 / \pi}$

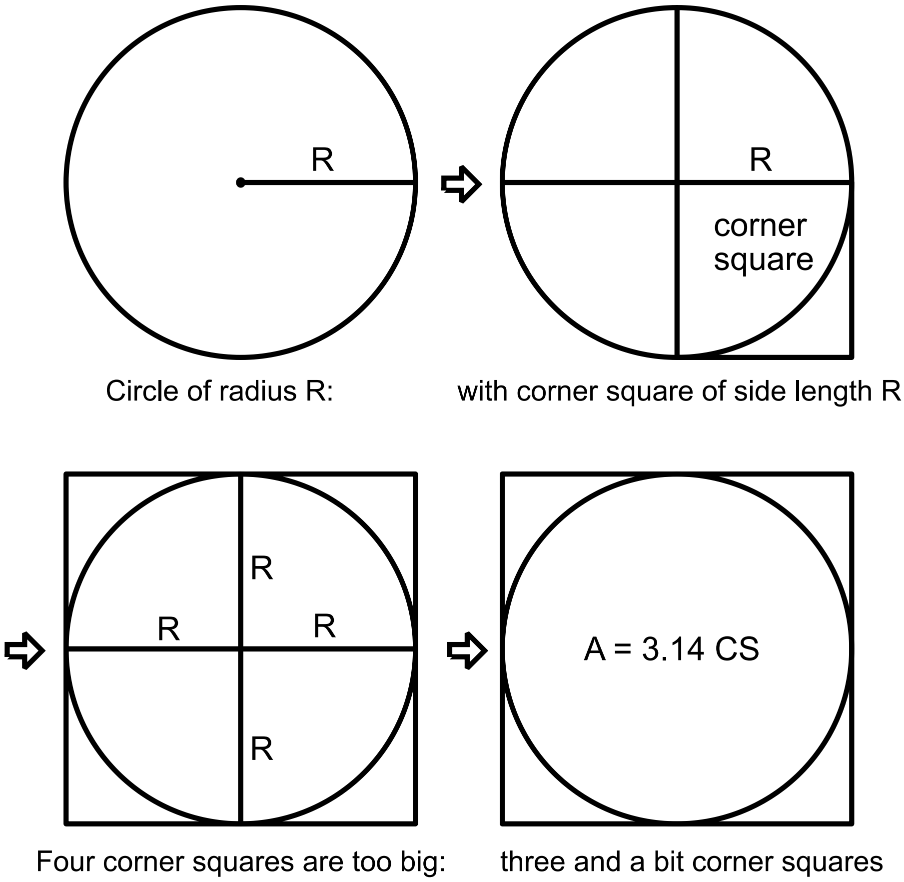
$R = 3.57\text{cm}$

After line 3, I declared “ignore the minus sign, kids”, the subtlety of why we do this utterly escaping my students. I knew (and so did they) there was mostly no hope of success – the algebra was beyond them, the symbolism was beyond them, they had little understanding or appreciation of the mathematical significance, power, history and beauty of π, the equation-solving steps beyond them.

The alternative pathway

Someone had shown me a different pathway – an empirical guess and check approach. What did students know? They knew the area of a square. So, build on that (constructivist learning theory). We drew a square, and we called this the corner square (CS) (see Figure 1). We could see the side lengths of the corner square were all R. How many corner squares would it take to cover the circle? They could readily see it was less than four. “Someone counted it very carefully and found it was just over 3. Three and a bit – very close to 3.14”. We now declared that to find the area of a circle we multiply the area of the corner square by 3.14. All my 8G students could follow this – it made visible sense to them. None of them could follow or make sense (at this stage of their learning journey) of the abstract version.

Figure 1. Showing the area of a circle is approximately 3.14 corner square areas



The conversation to support the new solution to the problem on Friday's test went like this.

Make a guess.	Maybe the radius is 2?
2×2 – What's that?	That's the area of the corner square!
Times by 3.14 - Why do that?	Cos that's how many corner squares there are!
Answer: 12.56	TOO SMALL because we are looking for an area of 40. Try a radius of 3?
3×3 – What's that?	That's the area of the corner square!
Times by 3.14 - Why do that?	*etc. etc…*

The language is simple, repetitive, and importantly within their understanding. We put the results into a table. As they worked, I noticed a strong connection with visual imagery that supported the process.

Radius	Area (aiming for 40)	Comments
2	12.56	too small
3	28.26	too small
4	50.24	too big – so the radius is between 3 and 4
3.5	38.46	too small – but getting close
3.6	40. 69	too big so it's between 3.5 and 3.6 which leads to…
3.56	39.80	too small – just
3.57	40.02	too big – only just and closest so far – to two decimal places!

Were my students ready to accept and use this approach? Not by a long way – they had become so conditioned to failure that positive entreaties from me had little effect initially. I think I never worked so hard as a teacher to encourage and support them. "It's on Friday's Test – the other Grade 8's don't know this method – I promise you we can all get this question on the test correct – we might not get other questions right but let's PLEASE give this one a go."

It did not happen easily but 'inch-by-inch' (sounds better than 'cm-by-cm') they started to get practice questions right. More importantly they could see the logic of the process. Then it seemed like the 'dam broke': "Hey – we can do these - they are easy!!"

To finish my personal experience with 8G, for Friday's test I appealed to my class to seek out and do this question first. Which they did and all smiled when the question appeared. The mathematics department collected statistics on how students performed on various test items. For this task 8A scored something like 95% correct as expected, 8B got 88% and progressively less success rates down through the grades – but 8G? 100% correct. I was called into the co-ordinator's office, thinking there would be excitement and welcoming of this beneficial approach which could advantage all students, not just my 8G. Not a word of support; the atmosphere was of consternation and more than a whiff of possible 'cheating'. I could not escape the feeling as I left the room that my students were not meant to get it correct! That there is a proper order and purpose to a maths curriculum, and I was uncomfortably disturbing its structure.

This personal experience happened many years ago. I have since often used and promoted the ideas and potential benefits, but it seems very little systemic progress has been made in adopting such empirical approaches into general use. Area of a circle is just one example; solving quadratic equations is another.

Lesson 2. Solving Quadratic Equations

In Australian schools, and I suspect in most other countries, significant time in Grades 9 and 10 is devoted to the study of quadratic equations and their solutions. Students are drilled to convert any quadratic equation such as $x^2 = 10 - 4x$ into the standard form of $ax^2 + bx + c = 0$, in this case, $x^2 + 4x - 10 = 0$. Why? Because in this form the three coefficients a, b and c can be easily read and plugged into that famous quadratic formula to get the solutions.

In my Grade 9 classes, confidence and mastery of this formula and its derivation was never strong. Indeed, it was quite the opposite: it induced negative reactions. For many students it was the 'beginning of the end of the road' of their mathematical journeys. It was too abstract but most importantly they clearly did not understand the logic of the rule. It was a 'black box' mystery.

I challenged my students (even the top classes) before using the formula to approximate an expected answer to $x^2 + 4x - 10 = 0$. Not a single student could offer anything – there is nothing in the appearance of the standard form to allow them any intuitive processing. So, I introduced a new standard form and an alternative solution method by rearranging $x^2 + 4x - 10 = 0$ into $x^2 + 4x = 10$ and then into $x(x + 4) = 10$. This format became a new standard form. "We are looking for a number, multiplied by another bigger by 4, to get an answer of 10."

All my Grade 9 students could follow this. They could 'read' in a way that made sense to them. Putting numerical guesses into a table quickly gives the positive solution, and surprisingly, the negative solution comes as a bonus. Students with somewhat better algebra can investigate if it always happens and why.

x	***x* + 4**	**product**	**comment**
1	5	5	too small
2	6	12	too big – the solution must be between 1 and 2
. . . .	. . . .	. . . .	
1.7	5.7	9.69	too small but close
1.75	5.75	10.06	too big but closest so far
1.76	5.76	10.13	just checking which is closest
1.74	5.74	9.9876	just checking which is closest – this one!
-5.74	-1.74	9.9876	interestingly, in finding the positive solution of 1.74 the table also provides the other solution which is -5.74.

This iterative method works for all types of quadratics. For example, $2x^2 + 5x - 11 = 0$ becomes $x(x + 2.5) = 5.5$ and solves just as easily. Even if the quadratic factorizes it soon yields its integer solutions. For example, $x^2 - 7x + 10 = (x - 5)(x - 2) = 0$ becomes $x(x - 7) = -10$ and then $x = 2$ or 5.

My Grade 9 students could follow every step. If they made an estimate error, they could recover, they could get the answer to any level of desired accuracy quickly and efficiently. They knew what they were working towards and were significantly more successful and positive about their learning than when using the formula.

Observed general features of this approach

As my students progressed, a few interesting and important learning features became prominent. Although these are informal observations from myself and other trial teachers, each of these seem worthy of serious discussion and analysis with teachers as they explore whether these numerical methods have long term potential at both a personal and systemic level.

i. **Mathematics.** This iteration is an utterly legitimate mathematical process. Indeed, in this electronic digital age many, if not a majority, of real industrial economic and scientific problems are being solved using iterative techniques.
ii. **Understanding**. There is a strong focus on the concept of the iterative process through what is essentially a first principles approach. Nearly every student could articulate to me what they were doing and why – at any stage of the process.
iii. **Access, Differentiation, Mixed Ability.** All students could make a guess and get started. The formula approach inhibits access unless the algebraic facility has been well established.
iv. **Language**. The presentation and discussion highlight the connection between everyday language and the language of mathematics.
v. **Does iteration take too long?** My students were solving a problem in about 3 minutes. That's about the same time the traditional approach takes, so it is not less efficient in terms of time.
vi. **Connections.** Because it is closer to a first principles approach than just using the formula, students were actively interconnecting ideas. For example, when finding the area of a circle, students connect ideas of area, algebra, and decimals.
vii. **Technology.** The effective and extensive use of calculators in this current era of STEM is to be encouraged.
viii. **Estimation**. As students began to achieve success, they became much better at making a 'next guess' estimate.
ix. **Mistakes.** You can make a mistake and recover. For example, making an estimate that is 'worse' than the previous one is soon noticed, and a better guess created.
x. **Discussion.** The process seemed to encourage beneficial small group work and associated discussion in a manner that does not occur in the formula approach.
xi. **Level of Accuracy.** Several of my students wanted to get the answers exact and worked out answers to 4 and 5 places (and then were disappointed when the calculator of the day could not 'go further').
xii. **Confidence.** Students became faster and more confident as they started to 'churn out' correct answers.

Obstacles and impediments to substantive systemic adoption

Over years of successfully exploring and enjoying these approaches, I have had much cause to ponder why they have not become more widespread, particularly at systemic policy levels.

Are numerical methods currently recognised and promoted within school policy and scope and sequence documents. It seems not. The current (2022) page of Wikipedia on quadratics ("Quadratic equation", 2022) has comprehensive coverage of solving quadratics but it reveals not a single word about numerical methods. Similarly, other web searches such as 'What is the fastest way to solve quadratics?' or 'What is the best way to solve quadratics?' all mention factoring, completing the square, using the classic formula, and graphing but not iterative techniques. Graphing calculators use iterative software, but this is often automated and hence the logic of the approach is often masked and not made explicit to students.

A second obstacle to adoption may be that there is some confusion of purpose for these tasks. For example, is the Friday test's area task (finding the radius for circle area 40 cm^2):

1. primarily about area and providing students with an algebraic tool to assist with the calculations?
2. primarily about building algebraic manipulation skills so the topic of area is merely a convenient context in the service of that algebra?
3. primarily to showcase the power and beauty of the number π?

It is probably parts of all three, but I contend that for struggling Grade 8 students these three are all largely unattainable. If the purpose is about area-related calculation then it seems to me a 'no-brainer' – many more students will benefit and be empowered by the iterative approach, gaining understanding and success. If the purpose is about the undoubted extraordinary beauty and usefulness of π, then Grade 8 is arguably not the place for this to happen for a majority of students.

Third, whilst 'business is booming', curriculum is not. The Australian Curriculum: Mathematics (v.9) (ACARA, 2022) is the formal policy provided to Australian schools to guide courses. A search reveals very little mention of iterative problem solving techniques anywhere in the Grade 8 to 10 guidelines and the two current textbooks I consulted did not present it in any way. While largely absent from school curriculum, it is not so in business and scientific endeavours. Any search will yield multiple references and scenarios to the widespread use of numerical algorithms. Sundaram (1998), for example, commented:

> *Optimization methods using differential calculus can be applied to solve certain problems but as the problem becomes too cumbersome then classical methods get replaced by iterative techniques. This process of finding solution iteratively involves extensive computations and there are several algorithms on iteration. (p. 1)*

A legacy of history and preserving the status quo

Why does $A = \pi R^2$ and associated problem solving feature in the Grade 8 curriculum for 13 year old students? Who put it there? Who decided that this is the appropriate level? Any survey reveals it to be enshrined in almost every Grade 8 textbook. One historical view is that before the 1940's and 1950's, Mathematics was in the liberal arts faculty at universities. It was not as connected with science fields as it is now, witnessed by the current growth of STEM subjects. Most students in schools studied arithmetic and were not engaged in higher mathematics. The small number that took higher mathematics were challenged to get into university by climbing a 'ladder' designed to 'weed out' students to find those worthy to enter the portals of a university. It was competitive, but participants willingly and knowingly took on the challenges. It was not a 'mathematics for all' course in any utilitarian sense. It seems at that time that tasks such as area of a circle were thought appropriate at the Grade 8 level.

Around the 1950's with school retentions growing rapidly and a realisation that mathematics beyond arithmetic was worthy of study for all students, new courses were needed. It seems in this transition that much content designed for a quite different cohort survived. It is arguable that the algebra skills needed for area or quadratics problems are too hard and too abstract at their current age levels for a significant number of students. Their negative experiences carry into adulthood and seem partly responsible for the mathematics anxiety and somewhat poor attitudes to school mathematics so evident in the wider community.

A somewhat depressing finale

Just recently, while preparing to finish this article a neighbourhood student sought my assistance in the following word problem which had been set for homework.

A rectangular floor vent is 12cm long and 6cm wide. It is to be enlarged by increasing the length and the width by x centimetres.

i. What is the new length of the floor vent (in terms of x)?
ii. Show that the area, A, of the new vent is given by $A = x^2 + 18x + 72$.
iii. The area of the new vent must be 50% more than the original area. Find the dimensions (to the nearest centimetre) of the new vent.

I found the task as presented to be rather depressing in two fronts.

First consider the context. Word problems are often used to provide contexts or scenarios to illustrate how mathematics is applied to 'real world' problems. But the scenarios are often spurious and the main purpose being to practise use of formulaic procedures. Here, no effort is made to establish a meaningful context. Presumably the question intends a vent for heating. For example, asking students to check their household to see if there are vents and of what size might establish the context as meaningful. The most common vent in house heating here is 30cm by 10cm which presumably is sufficient in size to provide appropriate volumes of air. A size of 12cm by 6cm is not at all common and probably insufficient for heating. Why does the author not account for this? The numbers are misleading and clearly contrived solely for the quadratic algebraic purpose, not for any meaningful problem to be solved. Pitman and Pateman (1985) in their report to the

Australian National Mathematics Curriculum and Teaching Program produced an overview of 'What's wrong with school mathematics?' by surveying teachers, students, parents, universities, and the wider community. At the top of their list was 'boring' closely followed by 'irrelevant'. The above type of word problem does little to alleviate these criticisms.

Second, consider the solution method. The expected explicit solution method is to generate a quadratic equation, to recognise the three coefficients, and then apply the quadratic formula. If the purpose was to solve the problem then this quadratic pathway is a very poor choice when the problem is solved numerically in a fraction of the time. If the purpose is to solve the problem (a big 'if'), then it yields quickly, efficiently, and logically using an empirical approach.

Size (cm)	Area (cm^2)	Comment
12 × 6	72	A 50% increase requires an area of 108
13 × 7	91	too small
14 × 8	112	slightly too large but this is the best to the nearest cm
13.9 × 7.9	109.81	greater precision if needed can then easily be found as required
13.8 × 7.8	107.64	just a little too small

Summary

Students need to be building a toolbox of solution methods and strategies and to learn to choose the appropriate tool for the task. In many tasks, such as those discussed here, the formulaic methods seem to be wholly inappropriate and the empirical method vastly superior. I finish with a hope embedded in a question. What needs to happen in curriculum planning to recognise the benefits of numerical approaches and to enshrine these in an appropriate manner into general systemic curriculum policy?

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About the Author

Charles Lovitt (clovitt@netspace.net.au), the winner of the 2021 ISDDE Prize, is a distinguished designer whose outstanding contributions to mathematics education over many years are characterised by their combination of depth and breadth. His work on the design and development of innovative materials for teaching and professional development has had a profound impact on the teaching and learning of mathematics, in Australia and far beyond. Focusing on teachers and the act of teaching, his efforts have always been about designing materials that support experiences that inform, challenge and support teachers in their day-to-day work. Charles' creativity in designing lessons that have the power to transform the learning of mathematics for many students sets him apart. Charles lives in Melbourne, Australia, but over his long career, has worked with teachers around the world.

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